Non-intrusive reduced-order modeling of the drift flux model using a residual recurrent neural network

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Summary. Projection-based Model Order Reduction (MOR) aims at reducing the computational cost associated with the solution of large-scale dynamical systems to be used in many-query settings such as optimization and control. For nonlinear systems, significant cost reduction is only possible through an additional approximation of the nonlinear terms to reduce the computational effort of the Reduced-Order Model (ROM). These hyper-reduction techniques often lead to instability when the nonlinear terms are not approximated with a high accuracy. Increasing the accuracy of the nonlinearity approximation increases the complexity of the ROM and will question the original motivation behind MOR to obtain a faster simulator and a system with lower number of states for controller design. In this study, a non-intrusive (data-based without the need for the physical model) Reduced Basis (RB) method is proposed for a highly nonlinear model, called the Drift Flux Model (DFM), to simulate multi-phase flow inside a pipe. A set of RB functions are extracted from a collection of high-fidelity solutions by changing the input signals of the system via a Proper Orthogonal Decomposition (POD). The solution of the ROM is obtained through a linear combination of these RB functions with coefficients obtained by a Residual Recurrent Neural Network (RRNN). The RRNN approximates the map between the input signals and the increment of projection coefficients of the high-fidelity solution onto the reduced space. The generation of the RB functions and the training of the RRNN are performed during the offline phase, while the RB solution of a new input signal can be recovered via the outputs of the RRNN in the online phase. The proposed method decouples the offline and the online phases, and provides fast and reliable solutions of the original DFM.

Problem description

One of the widely accepted models to simulate multi-phase flow is the DFM [1], a highly nonlinear set of conservation laws described as below:

$$\begin{cases} \dfrac{\partial}{\partial t}(\alpha_l\rho_l) + \dfrac{\partial}{\partial x}(\alpha_l\rho_l v_l) = 0, \\ \dfrac{\partial}{\partial t}(\alpha_g\rho_g) + \dfrac{\partial}{\partial x}(\alpha_g\rho_g v_g) = 0, \\ \dfrac{\partial}{\partial t}(\alpha_l\rho_l v_l + \alpha_g\rho_g v_g) + \dfrac{\partial}{\partial x}(\alpha_l\rho_l v_l^2 + \alpha_g\rho_g v_g^2 + p) = F + G, \end{cases} \quad t \in [0,T], x \in [0,L], \begin{cases} 1 = \alpha_l + \alpha_g, \\ v_g = K(\alpha_l v_l + \alpha_g v_g) + S, \\ p = (\rho - \rho_0)c_l^2 + p_0, \\ p = \rho_g c_g^2, \end{cases} \tag{1}$$

where $\alpha_i(t,x), \rho_i(t,x), v_i(t,x), p(t,x)$ represent volume fraction, density and velocity of phase i and the common pressure, respectively. The subscript $i \in \{l, g\}$ denotes the liquid and gas phase with c_i the sound velocity in the medium of phase i, K and S two constants implying the flow regime and p_0 and ρ_0 the reference pressure and density. Here, t represents time and T is the time horizon of the simulation. In addition, x denotes the spatial coordinate and L is the length of the spatial domain. Finally, $F(t,x)$ and $G(t,x)$, respectively, denote the frictional and gravitational terms, which add extra nonlinearity to (1).

Highly nonlinear finite-volume schemes are developed to solve (1) [1], rendering the discretized system of equations even more complex. Therefore, real-time simulations cannot be achieved unless powerful computational resources are available. Moreover, control design for such a complex system is generally infeasible. Hence, MOR should be applied.

Reduced-order model

Intrusive (projection-based) MOR of (1) leads to an unstable system unless the nonlinear parts are approximated with a high accuracy. To circumvent this issue, a non-intrusive (data-based) MOR is applied in this study, which in addition resolves the need to access the physical model and enables the use of highly nonlinear and accurate finite-volume schemes. The algorithm introduced in [3] is used here together with an RRNN structure as shown in Figure 1. The variables W and b are, respectively, the weight coefficients and the bias values of each node in the hidden layer, to be specified during the training. It is well-known that recurrent neural networks trained on the residual values (variation of states over each time step) have a higher capability in approximating dynamical systems [4].

The RRNN structure takes the input signals of the system and gives the temporal variation of the coefficients of the RB functions as an output. Since we are dealing with a dynamical system, the coefficients of the RB functions in the previous time step are also fed as an input to the RRNN.

Results

In the simulations for the RRNN, we have used one hidden layer consisting of 20 nodes with one time-step delay ($0:1$ in the delay layer means both $u(t)$ and $u(t-1)$ are considered as the training inputs). The delay in the recurrent structure

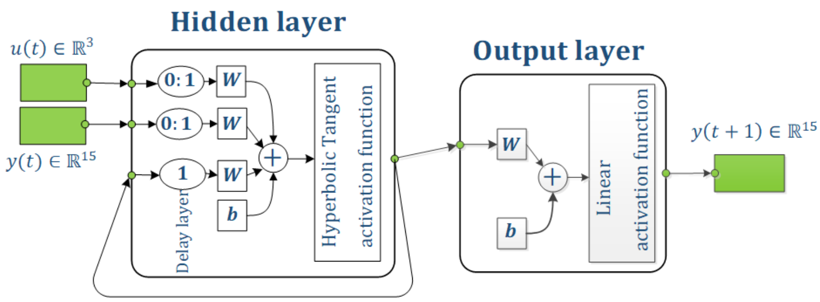

Figure 1: The nonlinear autoregressive network with exogenous inputs combined with RRNN.

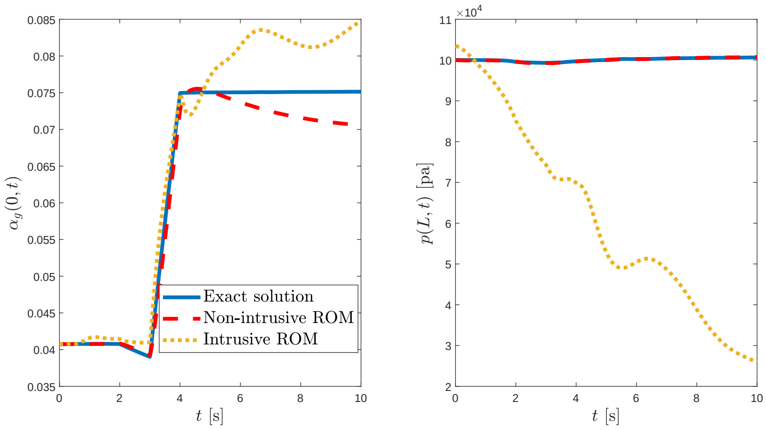

Figure 2: Comparison of the approximation of $\alpha_g(0,t)$ and $p(L,t)$.

is also set to one. The inputs to the system ($u(t)$ in Figure 1) are liquid and gas mass flow rates at the left boundary and the valve opening at the right boundary (3 inputs in total, $u(t) \in \mathbb{R}^3$). We have considered three independent variables α_g, v_l and p in (1) and assigned 5 RB functions for each (15 outputs in total, $y(t) \in \mathbb{R}^{15}$). To train the RRNN, five different samples of inputs $u(t)$ are fed into the structure and the coefficients of the hidden layer (W and b) are regulated to minimize the mean-squared error between the RRNN outputs and the temporal variations of the coefficients of the RB functions obtained after applying POD to the snapshots.

To test the RRNN generalization, a new input is provided for the network and the comparison of the state variables has been performed. The evolution of the state variables α_g at the inlet of the computational domain and the pressure p at the outlet of the computational domain is shown in Figure 2 for the actual solution, the intrusive and the non-intrusive ROMs. The intrusive method gives unbounded and unstable solution over time although the nonlinear terms are approximated by 10 collateral basis functions using the Empirical Interpolation Method [5]; two times more accurate than the linear terms. On the other hand, the non-intrusive one gives reasonably accurate results and is much faster than the full-order model. The speedup (obtained by dividing the CPU time of solving the full-order model to the CPU time of solving the ROM) for the non-intrusive method is 71.1 while for the intrusive one is only 2.05. To increase the accuracy in approximating the gas volume fraction, the number of the RB functions should be increased; however, as the approximation of pressure is of higher importance, we are satisfied with the performance of the non-intrusive ROM.

Conclusions

In this work, a non-intrusive MOR is applied to the DFM to reduce its corresponding computational time. Contrary to the projection-based MOR that develops an unstable system, the non-intrusive method provides an accurate and a stable reduced-order system, which is solved much faster compared to the original model.

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