Nonlinear damping in graphene resonators undergoing internal resonance

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Summary. We study the nonlinear damping of a nanomechanical graphene drum resonator. Laser interferometry is used to measure and optothermally actuate the resonator near an internal resonance condition. An unconventional nonlinear damping behavior is observed at the internal resonance frequency. A multimodal analytical model is constructed to understand the nature of this phenomenon and to relate the observed nonlinear damping to the physics of the system.

Introduction

Micro/Nano-mechanical systems are utilized in many technologies and often have been used for their sensing capabilities. An ideal framework for sensitive nanomechanical devices is 2-D materials, and especially graphene, due to its exceptional mechanical, electrical and thermal properties. By their atomically thin nature, these systems are fundamentally nonlinear. In addition to the geometric nonlinearities, graphene membranes also have nonlinear energy decay mechanisms[1]. Nonlinear damping in these devices is a fundamental limitation to their sensing capabilities yet its full understanding is an open question. Among different dissipation mechanisms, an important factor that is hypothesized to affect damping properties of graphene nanodrums is the intermodal couplings[2]. In this work, we study the nonlinear dynamics of a nanomechanical graphene resonator near its internal resonance condition to amplify the intermodal effects and uncover the physics between nonlinear damping and mode coupling.

Experimental method and observations

Experiments were conducted by optothermally actuating a graphene nanodrum while measuring its motion using laser interferometry. Optothermal actuation results in modulation of membrane tension, effectively creating a parametric excitation to the fundamental mode of the membrane that has an eigenfrequency ω_1 and direct excitation to a secondary mode of the membrane that has an eigenfrequency $\omega_2 \approx 2\omega_1$. We take advantage of this to investigate the effects of intermodal couplings and internal resonance on the nonlinear damping by exciting the membrane with an actuation frequency of $\omega_F \approx 2\omega_1$, which drives both modes resonantly. We experimentally study the membrane frequency response curves (Figure 1). By increasing the excitation level, we utilize the hardening type geometric nonlinearity of the system to match the internal resonance condition between these two modes. At the vicinity of internal resonance, we observe frequency "locking" at higher drive powers, where the increase in resonance peak with respect to the excitation amplitude is almost zero. We found that forcing the system even further breaks the "locking" barrier, causing a dramatic increase in amplitude and frequency of the parametric resonance, which we refer as "shooting".

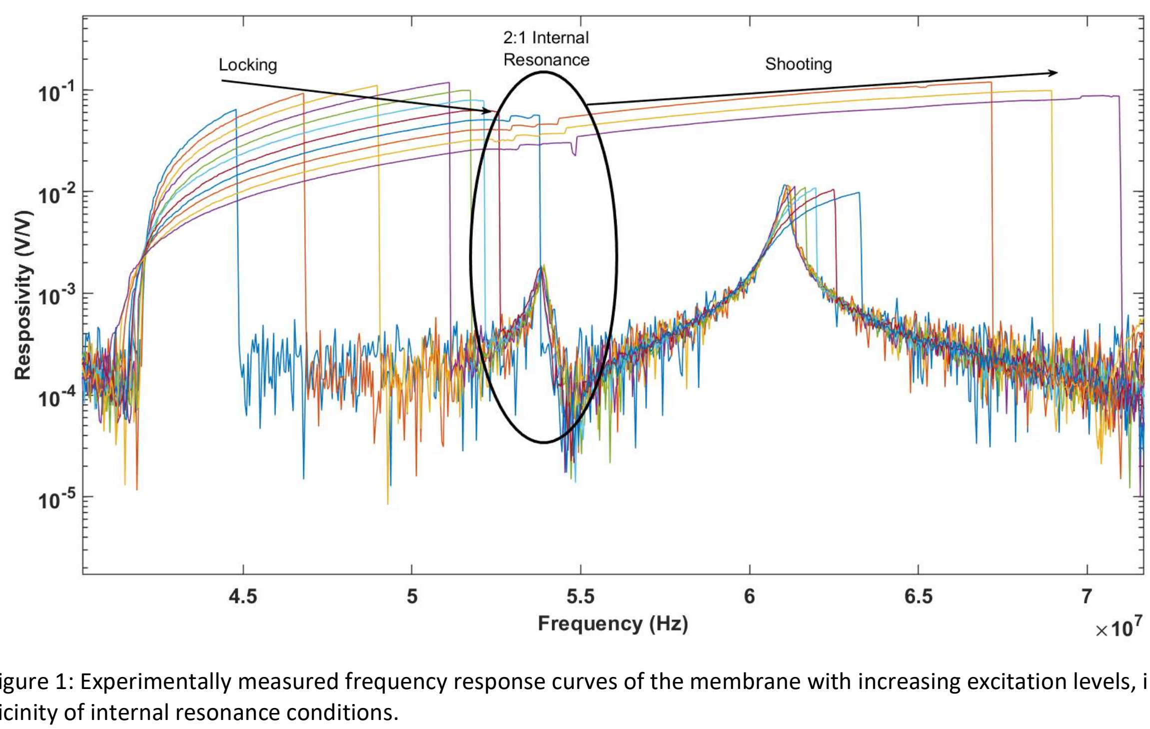

Figure 1: Experimentally measured frequency response curves of the membrane with increasing excitation levels, in the vicinity of internal resonance conditions.

Analytical model and results

Parametric resonance of nanomechanical systems is commonly modelled by using a single Mathieu-van der Pol-Duffing equation [3]. This model is only viable for nonlinear dynamic characterization far from the internal resonance as it will imply a varying intrinsic nonlinear damping for high driving powers in the vicinity of internal resonance, which has no physical foundations. This is clearly because of the intermodal effects, that necessitate multimodal modelling of the system. To capture the dynamics at the vicinity of internal resonance, we add to the Mathieu-van der Pol-Duffing single-mode model, a secondary linear oscillator that is directly driven by the excitation. We couple two modes by terms that arise from the nonlinear potential of 2:1 internal resonance $U_{cpl} = \alpha x_1^2 x_2$ (i.e., the coupling terms are: $-\partial U_{cpl}/\partial x_1$ for the primary mode, and $-\partial U_{cpl}/\partial x_2$ for the secondary mode):

$$\ddot{x}_1 + \omega_1^2 x_1 + \gamma x_1^3 + 2\alpha x_1 x_2 = F_1 x_1 \cos(\omega_F t) - 2\dot{x}_1(\tau_1 + \tau_{nl} x_1^2)$$
$$\ddot{x}_2 + \omega_2^2 x_2 + \alpha x_1^2 = F_2 \cos(\omega_F t) - 2\tau_2 \dot{x}_2$$

where γ is the Duffing coefficient, α is the intermodal coupling strength, F_1 is the parametric excitation level, F_2 is the corresponding direct forcing level and τ_l, τ_{nl} are the linear and nonlinear damping coefficients respectively. By analyzing the slow dynamics of the system, we reveal the modification of system parameters due to modal coupling, near the internal resonance condition ($2\omega_1 \approx \omega_2$). Using the intrinsic parameters of the modes for analysis, characterized by analyzing the uncoupled nonlinear response measurements, it is possible to see that the biggest effect of modal coupling is on the nonlinear damping. Additionally, the nature of the dramatic amplitude increase after the internal resonance is discovered by the bifurcation analysis of stationary solutions. The stationary (steady-state) solutions of the equations cease to exist in the vicinity of the internal resonance (see Figure 2) due to saddle-node bifurcation points. The annihilation of the bifurcation points connects two solutions branches, triggering the "shooting" phenomenon and can be used to characterize the coupling strength between the modes.

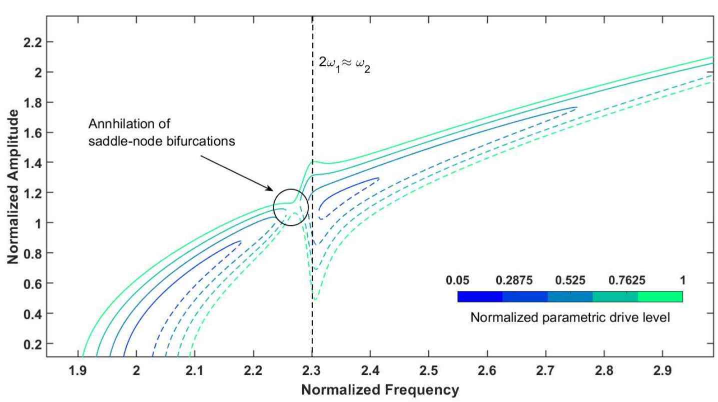

Figure 2: Steady state frequency response of the analytical model revealing the source of "shooting" behavior observed in the experiments, which is annihilation of saddle-node bifurcations.

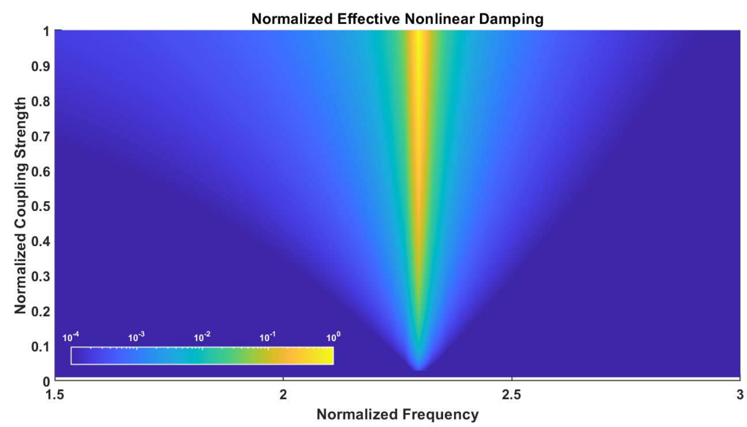

Figure 3: Effective nonlinear damping with respect to intermodal coupling strength and excitation frequency. Nonlinear damping is maximum at the internal resonance condition.

Conclusion

We report on nonlinear damping variation via 2:1 internal resonance in graphene nanomechanical resonators. We observe a massive increase in damping in the vicinity of internal resonance that is followed by a bifurcation causing a dramatic increase of amplitude and resonance frequency. To understand this phenomenon, the resonator has been modeled by a two-modes dynamical system undergoing a 2:1 internal resonance, which successfully explained the observations. This work shows a possible nonlinear dynamics methodology to characterize the intermodal coupling of nonlinear resonators by operating them in an internal resonance condition.

References

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