Tracing periodic solutions in noise-contaminated experiments

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Summary. In our study, we trace the steady-state solutions of a periodically forced nonlinear oscillator experimentally using control-based continuation. Our investigation is motivated by nonlinear aeroelastic oscillations where the noise load of experiments is often significant due to the unsteady flow in the wake of the vortex-shedding, oscillating bodies. Polluting our experiment with different amount of noise, we evaluate the resilience of the control-based continuation method against random perturbations and assess its capability to retain the response of the underlying deterministic system.

Introduction

Tracing a family of periodic solutions is a long-established area of studying nonlinear systems. For numerical dynamical models, the techniques of bifurcation analysis have become standardised and are available nowadays in several software packages. In experiments however, the steady-state periodic solutions are often traced by performing parameter-sweeps. Although this approach is usually easy to implement it provides limited information from the system as it can capture stable solutions only. Control-based continuation is a technique which implements the ideas of bifurcation analysis in experiments [1]. By applying feedback-control on the system, this method is capable to trace both stable and unstable steady-state periodic oscillations in an experimental setting. To achieve this, a control process is introduced which has to be stabilising and non-invasive, i.e. the steady-state solution of the controlled system is also solution of the open-loop system.

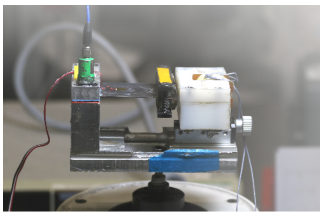
Our study is motivated by examples of aeroelastic oscillations originated in the interaction between inertial, structural and aerodynamic forces. The related vibrations, such as stall flutter of an aerofoil, caused by the periodic detachment and re-attachment of the flow, or the galloping oscillation of blunt elastic structures (e.g. cables) in airflow due to vortex-generation around the body, are substantially nonlinear phenomena. In the literature, several semi-empirical, low-degrees-of-freedom models were developed to investigate these oscillations [2]. Since these models use empirical coefficients there is a great demand for reliable parameter identification methods in this area.

Tracing steady-state solutions in noise-polluted experiments

In experiments, it may be challenging to obtain data which represent the dynamics of the investigated system accurately if the measurements subject to significant noise, as it is often the case with aeroelastic oscillations.

A nonlinear analysis of aeroelastic dynamical models can reveal a rich structure of different steady-state solutions and bifurcations [3, 4]. Nevertheless, in a physical experiment, these oscillations can result in a highly unsteady flow in the wake of the vortex-shedding objects. Thus, one may observe that the magnitude of measurement noise is comparable to the vibration amplitudes. In such conditions, the standard, open-loop parameter-sweeps may not be satisfactory for the characterisation of the system. For instance, in parameter regions with two co-existing stable steady-state solutions, the system may jump repeatedly between the two domains of attraction leading to an incomplete coverage of even the stable solutions [5]. The aim of our study is to demonstrate that tracing the solutions with a control-based method is more resilient against random perturbations than open-loop parameter-sweeps.

Case-study: parameter identification of a forced nonlinear oscillator under noise-load



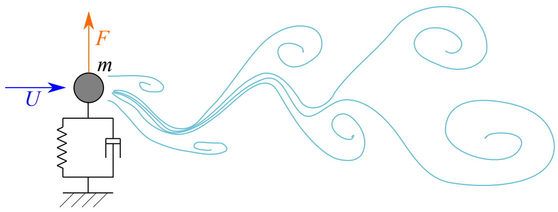


Figure 1: Left panel: The forced nonlinear oscillator. Right panel: The analogous model of vortex-induced vibrations.

To assess the capability of control-based continuation to deal with measurement noise we traced the steady-state solutions of a periodically forced nonlinear oscillator (see Fig. 1). In the experimental rig, the nonlinearity is primarily provided by the magnets mounted to the end of the elastic plate. As the plate vibrates the magnets are passing before the electromagnetic coil resulting in a nonlinear restoring force. The experiment can also be polluted with additional noise by driving current through the coil; thus, measurements can be conducted at different noise-levels.

This experimental setup may be seen as analogous to vortex-induced aeroelastic vibrations. These vibrations are similar to the galloping phenomenon in a sense that they can be originated in the vortex-shedding behind a blunt object as in the case of bridge cables. However, unlike in case of galloping where the frequency of vortex-shedding is equal to the

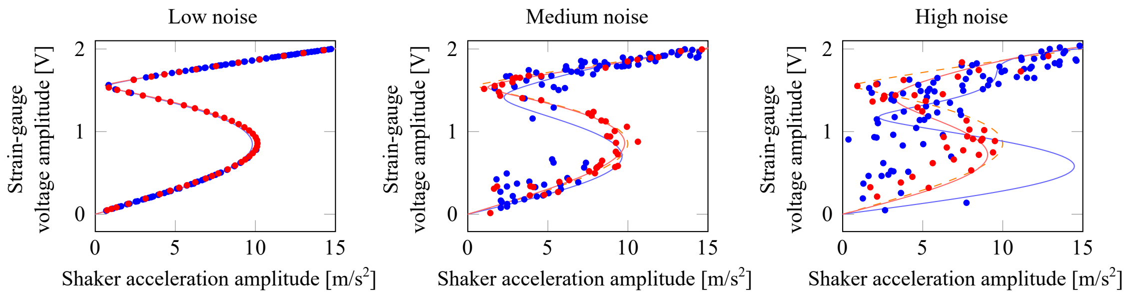


Figure 2: Steady-state response of the periodically forced nonlinear oscillator for different forcing amplitude. The blue markers correspond to amplitude-sweeps, the red ones to control-based continuation, respectively. The blue and red curves indicate the response of the fitted model while the dashed orange curve indicates the low-noise response in the panels corresponding to medium and high noise-levels.

vibration frequency, in case of vortex-induced vibrations, the frequency belonging to vortex-generation depends on the Strouhal number of the flow around the aerofoil [3]. Thus, while galloping is essentially considered as a self-excited vibration, vortex-induced vibrations are often modelled as forced nonlinear systems. One example is the semi-empirical model of Goswami et al. [6] given in the form

$$m\left(\ddot{x}+2\zeta\omega_{\rm n}\dot{x}+\omega_{\rm n}^2x\right)=F(x,\dot{x},\ddot{x},\omega t) \tag{1}$$

where m is the mass of the vibrating object, $\omega_{\rm n}$ is the structural natural angular frequency, ζ is Lehr's damping coefficient whereas the aerodynamic force F contains both the nonlinear and the periodic forcing terms.

Robustness against noise

We traced the S-shaped branch of periodic solutions of the experimental rig at different noise levels by performing both open-loop parameter-sweeps and control-based continuation. The acquired data was then used to identify the parameters of a one-degree-of-freedom Duffing-like oscillator with seventh-order nonlinearity. The results of parameter-sweeps and control-based continuation are compared in Fig. 2. At low noise level, the system can be characterised equally well by both methods. If a medium level of noise (around 25 % of the maximum vibration amplitude) parameter-sweeps begin to lose a part of the stable solutions as the jumps from low to high vibration amplitudes occur earlier than the folds in the low-noise solution branch. In the meantime, control-based continuation is still able to retain the original response reasonably well. However, at high noise levels (about 50 % of the maximum vibration amplitude) even the response curve fitted to the data obtained by control-based continuation visibly deviates from the branch corresponding to low noise. Nevertheless, control-based continuation still performs better as parameter-sweeps scarcely capture the bistable parameter region at this level of noise.

Conclusions

Comparing the response curves, acquired at different noise-levels, indicates that control-based continuation is more resilient against noise than open-loop parameter sweeps. Furthermore, having information about the unstable steady-state solutions of the system is clearly advantageous at nonlinear parameter fitting to noisy measurements. Therefore control-based continuation could be effectively used to investigate nonlinear aeroelastic systems experimentally.
Of course, our experimental rig has a relatively simple bifurcation diagram with only one bistable parameter domain and we also have access to the low-noise system. Consequently, if a more complicated structure of bifurcations is expected, based on deterministic theoretical models, the relationship between the measured and theoretical results may be less straightforward. Therefore, it is interesting to investigate what kind of response one can expect to find in a noise-contaminated measurement and how are the results obtained from the experiment related to the underlying deterministic system [7].

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