Nonlinear Dynamic Analysis of a Nonlocal Nanobeam Resting on Fractional Visco-Pasternak Foundation by Using the Incremental Harmonic Balance Method

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Summary. This paper investigates the dynamic behaviour of a geometrically nonlinear nanobeam resting on the fractional visco-Pasternak foundation and subjected to dynamic axial and transverse loads. The fractional-order governing equation of the system is derived and then discretized by using the single-mode Galerkin discretization. Corresponding forced Mathieu-Duffing equation is solved by using the incremental harmonic balance (IHB) method for the strong nonlinear case. Methodology and results are validated against the solution via multiple scales method for the weakly nonlinear case. A parametric study is performed for order two and three superharmonic resonance conditions and for primary resonance case. The results demonstrated a significant influence of fractional-order and damping parameter of the visco-Pasternak foundation as well as the nonlocal parameter and external excitation load on frequency response of the system.

Introduction

A special class of beam models is so-called nonlocal beams, where the nonlocal elasticity constitutive equation is employed to consider the small-scale effects [1]. Such nonlocal beams are usually referred to in the literature as nanobeams due to the nano-scale dimensions of structures. The incremental harmonic balance method is used to study nonlinear dynamic behaviour of beam structures by many authors, e.g. see [2]. The main advantage of harmonic balance techniques is that they can be employed to find the periodic solutions of strongly nonlinear systems without introducing a small parameter like in perturbation techniques [3]. Shen et al. [4] investigated the Mathieu-Duffing oscillator by the incremental harmonic balance (IHB) method and determined the stability of the periodic solution using the Floquet theory. Later, this method was extended to study the nonlinear Duffing [5] and forced Mathieu-Duffing type [6] fractional-order differential equations, where the fractional derivative term was approximated through the Galerkin procedure. This study aims to employ the incremental harmonic balance techniques to study the frequency response of a nanobeam system resting on the fractional visco-Pasternak type foundation. Pasternak elastic foundation model is usually used for materials which besides normal deflection contain shearing distortion. It should be noted that, under certain assumptions, a nonlocal beam model could represent nanostructures such as carbon nanotubes. In that case, boundary conditions to analyze the free or forced vibration of a nanobeam structure can be prescribed based on the end conditions in a carbon nanotube i.e. a number of layers of fixed atoms in the lattice (e.g. see [7]). If only one layer of atoms is fixed at both ends of carbon nanotube, we can use simply supported (S-S) boundary conditions in the mechanical model, and if several layers of atoms are fixed, we can use boundary conditions of clamped-clamped (C-C) nanobeam. The single-mode Galerkin method is used to discretize the governing equation and obtain the nonlinear response for the fractional-order forced Mathieu-Duffing equation. The results are verified by the comparison of amplitude-frequency curves from the multiple scales and incremental harmonic balance methods obtained for the superharmonic resonance conditions of order two and three and a primary resonance case.

Problem definition

The governing equation for the forced vibration of a nanobeam resting on the fractional visco-Pasternak foundation is derived based on the model presented in Fig.1. Following parameters are used: L is the length of the nanobeam, ρ is the density, A is the cross-sectional area of homogenous nanobeam, $\bar{\delta}$ and $\bar{G}_p$ are the coefficients of the fractional visco-Pasternak foundation, D^{α} is the operator of the Caputo fractional-order derivative, $\bar{F}_0$ is the amplitude of static load while $\bar{F}_1$ is the amplitude of the dynamic force of the frequency Ω_1.

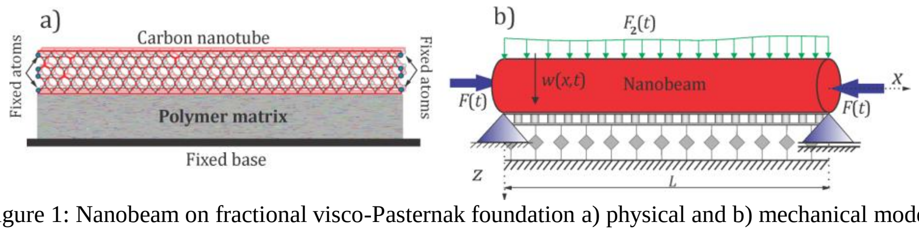

Figure 1: Nanobeam on fractional visco-Pasternak foundation a) physical and b) mechanical model

Based on the Euler-Bernoulli beam theory and von Kármán nonlinear deformation, nonlocal constitutive equation and the Newton's second law for the elementary part of the nanobeam, the following nonlinear fractional-order partial differential equation of motion of the nanobeam resting on the fractional visco-Pasternak foundation can be derived

$$\rho A\frac{\partial^2 w}{\partial t^2} + D^\alpha w\,\bar{\delta} + D^\alpha\left(\frac{\partial^2 w}{\partial x^2}\right)\bar{G}_p + (\bar{F}_0 + \bar{F}_1\cos\Omega_1 t)\frac{\partial^2 w}{\partial x^2}$$
$$-\frac{EA}{2L}\frac{\partial^2 w}{\partial x^2}\int_0^L \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 dx + EI\frac{\partial^4 w}{\partial x^4} - \mu\left[\rho A\frac{\partial^4 w}{\partial t^2\partial x^2} + D^\alpha\frac{\partial^2 w}{\partial x^2}\bar{\delta} + D^\alpha\left(\frac{\partial^4 w}{\partial x^4}\right)\bar{G}_p\right.$$
$$\left.+(\bar{F}_0 + \bar{F}_1\cos\Omega_1 t)\frac{\partial^4 w}{\partial x^4} - \frac{EA}{2L}\frac{\partial^4 w}{\partial x^4}\int_0^L \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 dx\right] = \bar{F}_2\cos\Omega_2 t\,. \qquad (1)$$

Solution procedures

The partial differential equation (1) is nondimensionalized and after introducing a new time scale $\bar{\tau} = \bar{\Omega}\tau$ and we assume the solution of nondimensional version of Eq.(1) as $\bar{w}(\bar{x},\tau) = \phi(\bar{x})q(\tau)$, nonlinear fractional-order forced Mathieu-Duffing equation is obtained in the following form

$$\bar{\Omega}^2 q'' + \varepsilon\gamma_{12}\bar{\Omega}^\alpha D_{\bar{\tau}}^\alpha q + (\omega_n^2 - \varepsilon\gamma_3 F_1\cos\bar{\tau})q + \varepsilon\gamma_4 q^3 = \tilde{f}\cos\bar{\tau}, \qquad (2)$$

For some initial guess q_0 of the steady-state modal amplitude, a neighbouring state of motion can be expressed in the form $q = q_0 + \Delta q$, $\bar{\Omega} = \bar{\Omega}_0 + \Delta\bar{\Omega}$, where q_0 and Δq can be represented as sums of trigonometric functions and corresponding weighting coefficients. Based on Galerkin procedure as described in papers [5, 6], Eq. (2) is discretised and Newton-Rapson method is applied to solve for increments of amplitude when $\Delta\bar{\Omega} = 0$.

Numerical results

Here, we show the numerical results obtained by the presented incremental harmonic balance (IHB) method for finding the frequency response of the system. We verify the results by comparing the steady-state frequency responses for the superharmonic resonance case $2\bar{\Omega} \approx \omega_n$ obtained by the IHB and the multiple scales method (MS), as given in Fig. 2a. Influence of the fractional-order derivative parameter for weak nonlinearity and the nonlocal parameter for strong nonlinearity on the amplitude-frequency responses are given in Fig. 2b and Fig. 2c, respectively.

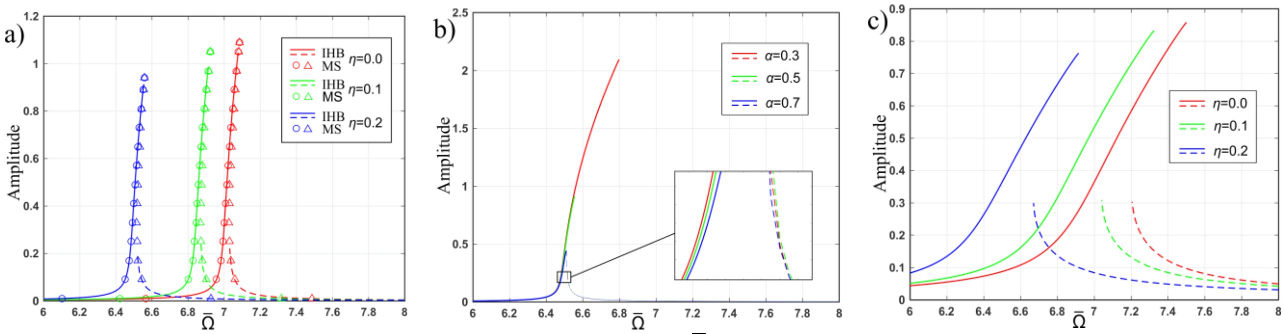

Figure 2: Frequency response for the superharmonic resonance case $2\bar{\Omega} \approx \omega_n$: a) weak nonlinearity, changes of nonlocal parameter, IHB vs MS, b) weak nonlinearity, changes of order of fractional derivative, c) strong nonlinearity, changes of nonlocal parameter.

Conclusions

From the validation study, it is revealed that the incremental harmonic balance method is in good agreement with the multiple scales analysis for the weakly nonlinear case. The advantage of the incremental harmonic balance method lies in the fact that it does not require the introduction of small parameter and thus strong nonlinearity cases can be observed. It has been demonstrated that introduction of the incremental harmonic balance method in the analysis of nonlocal structures can possibly lead to more reliable analysis of strongly nonlinear nano-scale systems.

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