Nonlinear Dynamics of a Ring-based Vibratory Energy Harvester

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Summary. This paper is concerned with nonlinear dynamic analysis and design of a novel ring-based Bi-stable energy harvesting device that is considered as an alternative to the beam and tube models used thus far. The Mathematical model for the ring structure to generate nonlinear harvester model as well as the nonlinear magnetic force that acts on the ring structure is formulated. The highly sensitive ring second flexural mode when combined with the nonlinear harvester model as well as nonlinear external magnetic force results in an ideal combination that yields increased frequency range. Numerical predictions of nonlinear dynamic response characteristics when the system is subjected to ambient harmonic excitation have been performed for the purposes of gaining an insight into the dynamics and power generation of this new class of harvesters.

Concept and Modeling

In this paper, the nonlinear governing equations of flexural motion of vibrating thin circular rings are developed for the purposes of investigating the nonlinear dynamic behavior of ring harvester. Galerkin's procedure is used to discretize the nonlinear equations for numerical response predictions. Gebrel et al [1] have presented the first study of a novel ring-based mono-stable energy harvesting device employing a linear system model subjected to ambient as well as nonlinear magnetic forces. The geometry and parameters used in the present study have been described in detail in [1].The general equations of motion that govern the transverse and circumferential motions are derived via Hamilton's principle, as described in [2]. In the present study, models using various configurations for the magnets are examined so that efficient bi-stable energy harvesting systems utilizing the sensitive second mode of a ring structure may be realized in practice. A schematic diagram of the magnetic configurations system is shown in Figure 1. In order to represent the oscillatory nonlinear magnetic force that acts on the ring structure, a novel design and analysis of a theoretical model formulation is employed. This analysis is restricted to mono-stable/bi-stable configurations that depend on the nonlinear terms. Magnets B and C are considered identical and their distance from magnet A is designated as d as shown in Figure 1.

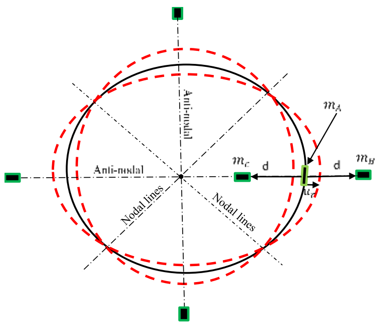


Figure 1: A schematic of ring and magnet configurations

The present study focuses on bi-stable energy-harvesting based on ring structure using the primary and secondary co-ordinates. System equations (1) and (2) have been employed together with equation (3) that represents the output power generation. The final nonlinear equations of motion that govern the nonlinear dynamic behavior of ring harvester employing the second flexural mode with nonlinear magnetic force as well as harmonic ambient excitation are derived as

$$\left[\rho h\pi + 2\rho h\pi\left(\frac{n\gamma}{2r}\right)^2 A_n^2\right]\ddot{A}_n + 2\rho h\pi\left(\frac{n\gamma}{2r}\right)^2 A_n B_n \ddot{B}_n + 2\zeta\omega_0\dot{A}_n + \left[\frac{EI}{br^4}(n^2-1)n^2 + k_r\right]\pi A_n +$$

$$\left[\frac{EA}{br^2} + k_r\right]\left(\frac{n\gamma}{2r}\right)^2 [A_n^2 + B_n^2]\pi A_n + 2\rho h\pi\left(\frac{n\gamma}{2r}\right)^2 [\dot{A}_n^2 + \dot{B}_n^2]A_n - \gamma_1 \underline{I} = f_{Nm1}(A_n, B_n, \theta_i) + f_e \quad (1)$$

$$\left[\rho h\pi + 2\rho h\pi\left(\frac{n\gamma}{2r}\right)^2 B_n^2\right]\ddot{B}_n + 2\rho h\pi\left(\frac{n\gamma}{2r}\right)^2 A_n B_n \ddot{A}_n + 2\zeta\omega_0\dot{B}_n + \left[\frac{EI}{br^4}(n^2-1)n^2 + k_r\right]\pi B_n +$$

$$\left[\frac{EA}{br^2} + k_r\right]\left(\frac{n\gamma}{2r}\right)^2 [A_n^2 + B_n^2]\pi B_n + 2\rho h\pi\left(\frac{n\gamma}{2r}\right)^2 [\dot{A}_n^2 + \dot{B}_n^2]B_n = 0 \quad (2)$$

$$L\dot{\underline{I}} + \check{R}\,\underline{I} + \gamma_1\dot{A}_n = 0\,. \quad (3)$$

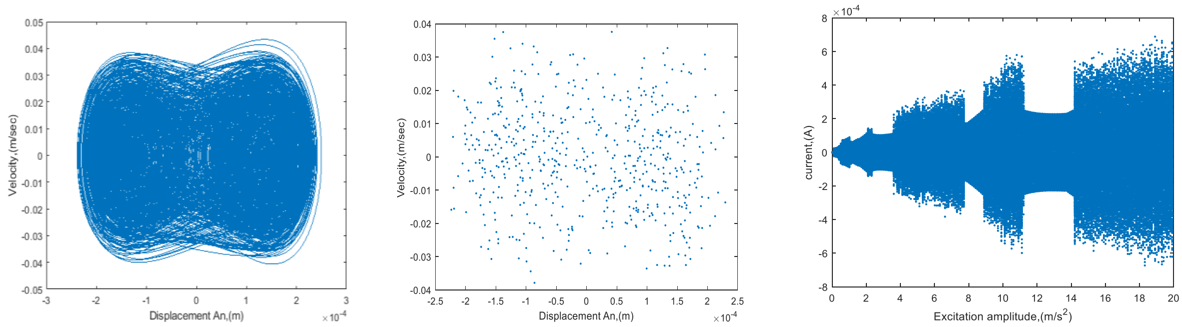
The parameters A_n and B_n represent the displacement of the ring in the transverse primary and the secondary directions, E is the Young's modulus, I denotes area moment of inertia for the ring cross-section , ρ is the mass density, and EI denotes flexural rigidity. The quantification of the nonlinear terms are governed by the parameter γ . Also, A is the cross sectional area of ring, b the axial thickness of ring, h the radial thickness, r the mean radius of the ring, ζ the mechanical damping ratio while ω_0 and n , respectively, represent the natural frequency and the number of modes. Oscillatory external nonlinear magnetic force magnitude is represented by $f_{Nm}\,(A_n, B_n, \theta_i)$, while the area moment of inertia of the ring cross section about its neutral axis is expressed as $I = bh^3/12$. The harmonic excitation to be received from the ambient vibratory energy sources is represented by $f_e = f cos(\omega t)$, where ω is the excitation frequency, and f is the excitation amplitude. The positions of magnets on the system correspond to θ_i , $i = 1$,2, 3,4. Induced electrical current is denoted by $\underline{I}$, L is the inductance of the coil, and $\breve{R}$ represents the load resistance, while γ_1 denotes the transducer constant [3]. The expressions for the nonlinear magnetic force that affects the system at four positions are derived in the primary co-ordinate A_n as

$$f_{Nm1}(A_n, B_n, \theta_i) = \frac{\mu_0}{2\pi} M_A M_B V_A V_B \sum_{i=1}^{4}\left(\cos(n\theta_i) - \frac{n\gamma}{2r}A_n\right) * \left[\frac{3}{\left(d - A_n\cos(n\theta_i) - B_n\sin(n\theta_i) + \frac{n\gamma}{4r}[A_n^2 + B_n^2]\right)^4} - \frac{3}{\left(d + A_n\cos(n\theta_i) + B_n\sin(n\theta_i) - \frac{n\gamma}{4r}[A_n^2 + B_n^2]\right)^4}\right], \quad (4)$$

where M_A, M_B are the magnetization, V_A, V_B represent the volume of the source magnet, and d is the distance between magnets.

Results and Discussion

In the present study, a nonlinear model which includes a complex nonlinear inertia/stiffness terms as well as a nonlinear magnetic force as depicted in equations (1) and (2) have been employed. For the purposes of predicting the nonlinear response characteristic of the bi-stable ring harvester, equations (1) and (2) have been solved numerically. The system parameters are chosen based on the available experimental set up that has been used to investigate the system natural frequency in the previous study [1]. The details regarding the electrical subsystem have been described in detail in ref [1]. Figure 2(a) shows the phase-plane trajectory for bi-stable behavior depicted via two-well potential when the harvester is under ambient excitation and excitation frequency of $60\, rad/sec$. The nonlinearities seem to be evident from the plot, hence suggesting the behavior of the bi-stable configuration for the harvester due to nonlinearities of the system as well as nonlinear magnetic force. Furthermore, nonlinearity can be seen in the Poincare' map results as shown in Figure 2(b), where the map appear as a cloud of unorganized points in the phase plane in Figure 2(a) due to the influence of nonlinear terms. Figure 2(c) shows the bifurcation diagram of the response current under various excitation levels. The excitation level ranging from $0 - 20\ m/s^2$ has been considered for this study. It may be inferred from this Figure that the bi-stable harvester undergoes alternating periodic and chaotic responses with the increase of the excitation level, which demonstrates a strong nonlinear behavior.



(a) (b) (c)

Figure 2: (a) Two well-potential bi-stable harvester, (b) Poincare' map for bi-stable harvester, and (c) Bifurcation diagram of response versus excitation amplitude for bi-stable harvester.

Conclusions

The main objective of this paper is to present the feasibility as well as nonlinear dynamic characteristics of a ring-based vibratory bi-stable energy harvester. When compared with the corresponding mono-stable harvester, an increased frequency range have been demonstrated. The dynamics of this class of harvester has been examined via dynamic response, Poincare map as well as bifurcation plots. The results provide confidence in employing the inherent bi-stability behavior available in this class of energy harvesters for energy production, in practice.

References

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