Forced vibration of spring pendulum with nonlinear springs connected in series

Jan Awrejcewicz*, Roman Starosta **, Grażyna Sypniewska-Kamińska **
* *Technical University of Łódź, Department of Automatics and Biomechanics, Łódź, Poland*
** *Poznań University of Technology, Institute of Applied Mechanics, Poznań, Poland*

Summary. An attempt to solve the problem and to conduct a qualitative analysis of the forced vibration of the spring pendulum containing nonlinear springs connected in series is made in the paper. The method of multiple scales in time domain (MMS) has been employed in order to carry out the analytical computations. The MMS allows one, among others, to predict the resonances which can appear in the systems. The approximate solution of analytical form has been obtained for vibration at main resonance.

Introduction

Elastic elements arranged in various kinds of connections (in serial, in parallel or in branching) are widely applied in many mechanisms, mechatronic devices and more and more often in micromechanical systems [1], [7]. When the massless springs are connected in series or in branching, modelling of them as massless elastic links, commonly used in discrete approach, leads to the mathematical model equations among which there are algebraic equations beside the differential ones. The algebraic equations describe then the equilibrium of the nodes at which the springs connect with each other. In the linear case, spring connections of such types create no greater difficulties. Depending on the degree of complexity of the connection, the equivalent spring constant can be introduced or one can retain in the mathematical model the algebraic equations that are linear, which results in a positive semi-definite mass matrix [2]. In nonlinear systems, the principle of superposition does not apply, which is a source of certain computational difficulties. It should be emphasized the serial springs connections increase the whole system pliancy, thus its nonlinear character manifests itself even more.
Various nonlinear oscillators with serially connected springs were investigated by many authors mostly numerically. An approximate analytical approach to the free vibration of oscillators with two nonlinear springs or one linear and another nonlinear spring is used among other in papers [4-5] and [3], [6] respectively. The free vibration of two mechanical systems with springs connected in series is studied using MSM in [5].

Mathematical model

The pendulum with two serially connected springs, presented in Fig. 1, is constrained to the motion on the vertical plane. Z_1 and Z_2 stand for the total elongation of the springs whereas L_{0i} denotes the length of the ith non-stretched spring. The springs nonlinearity is of the cubic type, i.e. $F_i = k_i(Z_i + \Lambda_i Z_i^3)$ for $i = 1,2$, and the nonlinear contributions to the whole elastic force are assumed to be small. There are two purely viscous dampers in the system. The system is loaded by the torque of magnitude $M(t) = M_0 \cos(\Omega_2\, t)$ and by the force $\mathbf{F}$ whose magnitude changes also harmonically i.e. $F(t) = F_0 \cos(\Omega_1\, t)$. Although the system has two degrees of freedom, its state is unambiguously determined by three time functions: the elongations Z_1 and Z_2 and the angle Φ.

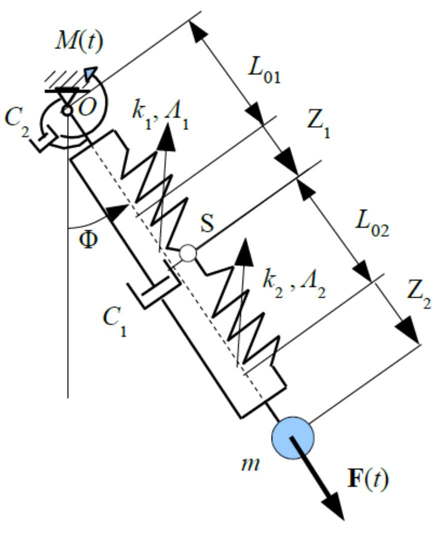

Figure 1: Forced and damped spring pendulum with two nonlinear springs

Two equations of motion, obtained using the Lagrange formalism, and the equilibrium equation of the join S govern the dynamic behaviour of the pendulum. They are as follows:

$$\left(1+\frac{dZ_1}{dZ_2}\right)^2 (m\ddot{Z}_2 + C_1\dot{Z}_2) + m\left(1+\frac{dZ_1}{dZ_2}\right)\dot{Z}_2^2\ddot{Z}_1 + k_2 Z_2(1+\Lambda_2 Z_2^2) +$$
$$k_1 Z_1(1+\Lambda_1 Z_1^2)\frac{dZ_1}{dZ_2} - m\left(1+\frac{dZ_1}{dZ_2}\right)\left((L_0+Z_1+Z_2)\dot{\Phi}^2 + g\cos\Phi - f_0\cos(\Omega_1 t)\right) = 0, \tag{1}$$

$$m(L_0+Z_1+Z_2)^2\ddot{\Phi} + C_2\dot{\Phi} + 2m(L_0+Z_1+Z_2)\left(1+\frac{dZ_1}{dZ_2}\right)\dot{Z}_2\dot{\Phi} +$$
$$mg(L_0+Z_1+Z_2)\sin\Phi - M_0\cos(\Omega_2 t) = 0, \tag{2}$$

$$k_2 Z_2(1+\Lambda_2 Z_2^2) = k_1 Z_1(1+\Lambda_1 Z_1^2), \tag{3}$$

where: $L_0 = L_{01} + L_{02}$.
Equations (1) – (3) are supplemented by the initial conditions of the following form

$$Z_1(0) + Z_2(0) = Z_0,\ \ \dot{Z}_1(0) + \dot{Z}_2(0) = v_0,\quad \Phi(0) = \Phi_0,\quad \dot{\Phi}(0) = \omega_0, \tag{4}$$

where $Z_0,\ v_0, \Phi_0, \omega_0$ are known quantities.
The derivative $\frac{dZ_1}{dZ_2}$ one can calculate taking into account Eq. (3) which simplifies significantly the governing equations. The method of multiple scales in time domain with three time variables is applied to solve the considered problem. Because of the algebraic-differential character of the motion equations the use of the method requires an appropriately modified approach. Omitting the details of the adaptation, it is worth noting that at each subsequent approximation there is an additional algebraic equation that after differentiating allows one to determine the relationship between the derivatives of the mutually dependent coordinates Z_1 and Z_2. The approximate solution obtained allowed for prediction of the resonance conditions. Then, the governing equations have been modified appropriately in order to describe the main resonance. The solution to the resonant vibration problem has semi-analytical form, because of the equations of modulation of the amplitudes and phases are solved in numerical manner.

Conclusions

The dynamics of the 2-dof system containing two serially connected nonlinear springs has been studied. The mathematical model consists of the differential and algebraic equations. The approximate solution to the governing equations, up to the third order, has been obtained using MMS with three time scales. The forced vibration of the pendulum have been analysed for two cases: far from resonance and in the resonance conditions. The analytical or semi-analytical form of the solution is the main advantage of the applied approach giving the possibility of the qualitative and quantitative study of the pendulum dynamics in wide spectrum.

Acknowledgments

This paper was financially supported by the grant of Ministry of Science and Higher Education 02/21/DSPB/3544.

References

[1] Andrzejewski R., Awrejcewicz J. (2005) Nonlinear Dynamics of a Wheeled Vehicle. Springer, Berlin.
[2] Bishop R.E.D, Gladwell G.M.L, Michaelson S. (2008) The matrix analysis of vibration. Cambridge University Press, London.
[3] Lai S.K., Lim C.W. (2007) Accurate approximate analytical solutions for nonlinear free vibration of systems with serial linear and nonlinear stiffness. *J. Sound Vib* **307**:72-736.
[4] Sanmiguel-Rojas E., Hidalgo-Martinez M., Jimenez-Gonzales J.I., Martin-Alcantara A. (2015) Analytical approaches to oscillators with nonlinear springs in parallel and series connections. *Mech Mach Theory* **93**: 39-52.
[5] Starosta R., Sypniewska-Kamińska G., Awrejcewicz J. (2017) Quantifying non-linear dynamics of mass-springs in series oscillators via asymptotic approach. *Mech Syst Signal Pr* **89**:149 – 158.
[6] Telli S., Kopmaz O. (2006) Free vibrations of a mass grounded by linear and nonlinear springs in series. *J. Sound Vib* **289**: 689-710.
[7] Wang Y.J., Chen C.D., Lin C.C., Yu, J.H. (2015) A nonlinear suspended energy harvester for a tire pressure monitoring system. *Micromachines* **6**: 312 – 327.