Drag Forces in Non-Uniform Cantilever Beam Oscillating in Viscous fluid

Lalsingh Devsoth and Ashok Kumar Pandey
Indian Institute of Technology Hyderabad, Kandi, Sangareddy, Telangana,502285.

Summary. Sensitiveness of the micro-mechanical cantilever beams strongly depends on the fluid damping such as damping due to drag and squeeze film. There have been many studies associated with drag force computation in uniform cantilever beam oscillating in isolation. In this work, we are presenting drag force computation in non-uniform cantilever beam with linear and quartic converging and diverging widths. Finally, we find the variation of added mass effect and quality factor based on drag force as a function of degree of non-uniformity, i.e., tapering parameter. It is found that the quality factor increases for converging beam and decreases for diverging beam. The effect is found to be more in quartic varying beam.

Introduction

Micro-Electro-Mechanical Systems (MEMS)/ Nano-Electro-Mechanical Systems (NEMS) such as biosensors, atomic force microscope, etc., generally operate in fluid. Therefore it is essential to compute drag forces due to fluid and structure interaction [1]. To compute drag forces in uniform cantilever beam, there exist formulas to compute fluid damping and added mass effect of a rigid circular cylinder oscillating with certain frequency in air [2]. To generalize the formula for arbitrary cross-section, Tuck [3] had developed formulation to compute drag forces and added mass effect. Although, generalized formula is written in terms of unknowns pressure and vorticity it appears little complex. However, he also extended it to a thin ribbon having zero thickness. To further include the effect of mode shape and rectangular section, Sader [4] modified hydrodynamic function for cantilever beam with rectangular section by modifying the hydrodynamic function of circular cylinder given in [2]. Zhang et al. [5] obtained more explicit form of drag force and add-mass effect in cantilever beam. However, all the previous study are limited to uniform cantilever section with circular or thin-rectangular sections. In this study, we compute the drag forces in non-uniform cantilever beam based on the mode-shapes and frequencies of corresponding converging and diverging beams and compare it with that of uniform beam.

Theoretical model for non-uniform beam

The oscillations of a cantilever beam in a viscous fluid is governed by a linearized unsteady Stokes equation $-i\omega\rho\mathbf{v} = -\nabla p + \mu\nabla^2\mathbf{v}, \nabla.\mathbf{v} = 0$. Fourier transformed unsteady Stokes equation is solved using the stream function [3], where, ω frequency of oscillation, velocity $\mathbf{v}$ and pressure p is obtain by transforming above equation into vorticity equation and pressure harmonic equation $\nabla^2 p = 0$ [3]. Assuming the velocity of vibrating beam in z-direction with velocity, $u = u_0 e^{i\omega t}$[1], where, u_0 is the maximum amplitude of the vibration, the hydrodynamic function is found by integrating pressure along beam width in terms of Reynold's number ($\mathtt{Re} = \frac{\rho\omega \mathtt{b}^2}{\mu}$, where, b is the width of beam) [3]. Subsequently, damping coefficient at each section of the beam is found from imaginary component of hydrodynamic function and then the modal damping. To compute the force for a thin beam with converging and diverging widths, we use the corresponding frequencies and mode shapes obtained by Sajal et al.[7]. Here, the beam width is taken as $b = b_0(1 + \alpha\frac{x}{L})^n$, where,$n =$ 1(linear),$n = 4$(quartic), α is tapering parameter varying from -0.5 to 0.5. For converging beam, it is negative and for diverging beam it is positive. The mode shape of non-uniform beam is given by $w(x) = \frac{v(x)}{\sigma(x)}$, where, $v(x)$ is the exact mode shape of the uniform beam, $\sigma(x)$ is function to capture non-uniform effect which is taken for linear variation in width as $\sigma(x) = \sqrt{1 + \alpha\frac{x}{L}}$ and for the quadratic as $\sigma(x) = (1 + \alpha\frac{x}{L})^2$ [7]. The area and area moment of inertia of the beam section also vary with width. The different modal frequencies $\omega = (\frac{\lambda}{l})^2\sqrt{\frac{EI}{\rho_b A}}$ of the non-uniform beam are obtained by solving the characteristic equation of the beam [7, 6], where, λ for linearly tapered beam are 4.3119, 4.0978, 3.922, 3.7710, 3.6420, 3.5160, 3.4129, 3.3208, 3.2368, 3.162, 3.0958 for tapering parameter are -0.5, -0.4, -0.3, -0.2, -0.1, 0, 0.1, 0.2, 0.3, 0.4, 0.5. The corresonding λ for quartic case are 7.558, 6.2981, 5.3361, 4.5882, 3.9940, 3.516, 3.1241, 2.7989, 2.5259, 2.294, 2.0961. Finally, quality factor (i.e $\mathtt{Q} = \frac{\mathtt{mass}\times\omega}{\mathtt{C_{davg}}}$) and the modal damping coefficient $C_{davg} = \frac{\int_0^L c_{dx} w^2(x)dx}{\int_0^L w^2(x)dx}$ are computed.

Results and Discussion

For a non-uniform microbeam as shown in Figure 1(a) undergoes vibration in ambient air of viscosity of $1.8 \times 10^{-5}\frac{kg}{m-s}$) which induces fluid damping and added mass. To compute these quantities, we take a microbeam of density $\rho =$ $2300 kg/m^3$, elastic modulus, $E = 62.5$Gpa, width at fixed end as $b_0 = 40\mu m$, length $L = 200\mu m$, and thickness $h_0 = 0.965\mu m$. To compare the influence of drag forces and added mass, we plot the hydrodynamic function per velocity for free end width corresponding to tapering parameters of ($\alpha = -0.5, 0, 0.5$) as shown in Fig 1(b)-(e) for both linear and quartic converging diverging beam. Figure 1(b) and Fig 1(c) represent the added-mass and damping effect, respectively, for a linear converging diverging beam. The trend of both the curve show decrease in their value as Re increase. Similarly, for the quartic converging diverging beam, Fig 1(d) and Fig 1(e) show that the added-mass and damping effect similar trend. However the effect is more pronounced in quartic beams. For validation, we have also compared the results of uniform beam ($\alpha = 0$) results with circular cylinder case mentioned in [2]. Subsequently, for frequency variation as shown in Fig 1(f) from [7], Fig 1(g) and Fig 1(h) show variation of damping coefficient and added-mass of non-uniform beam with different tapering parameters for linear and quartic varying beams. As the tapering ratio increases,

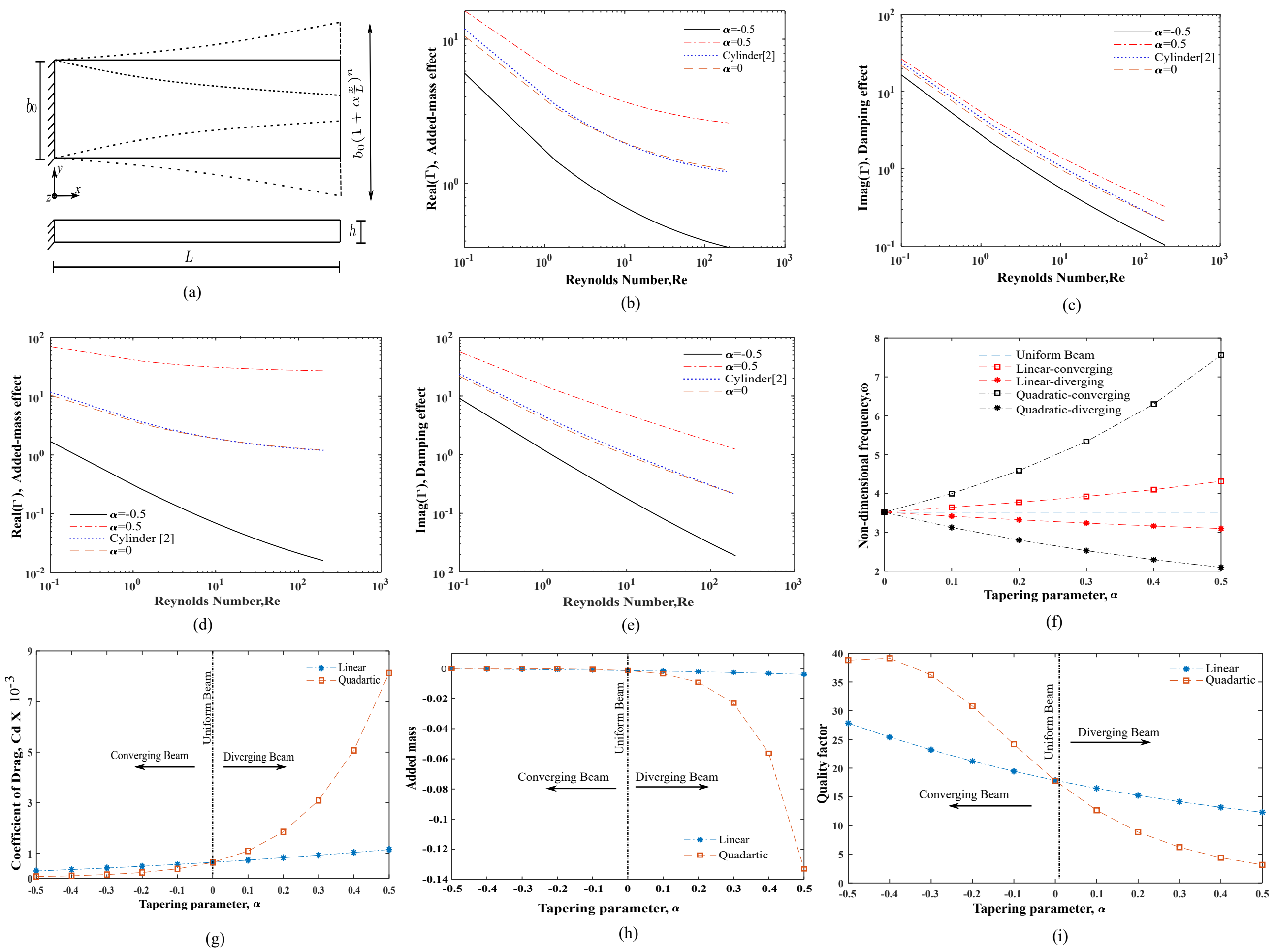

Figure 1: (a) A typical cantilever beam; Variation of hydrodynamic function with Re for $\alpha = -0.5, 0, 0.5$ to capture (b)added-mass and (c)damping coefficient of linearly varying beam, and (d) added-mass and (e) damping coefficient of quartic varying beam. Variation of (f) non-dimensional frequency, (g) coefficient of drag, (h) added mass, (i) Quality factor of converging-diverging beam with tapering ratio α.

overlaping/wet area increases which lead to increase in damping and reduction in added mass effect. Finally, the variation of quality factor for liearly and quartic beam are shown in Fig 1(i). It shows that quality factor increases for converging beam and decreases for diverging in both types non-uniform beam. The results presented here will be useful in designing non-uniform beam based mass sensors [8].

Conclusions

We have computed damping coefficient and added mass effect due to drag forces in non-uniform cantilever beam based on semi-analytical method. For free-end width of non-uniform beam corresponding to $\alpha = -0.5$, 0, 0.5, we compare the hydrodynamic functions corresponding to damping and added mass effect per unit length. Subsequently, we find modal damping coefficient and modal added mass effect linear and quartic varying beam with $\alpha = -0.5$, 0, 0.5. It is found that damping effect increases with increases in tapering parameter. The corresponding quality factor increases with decrease in tapering parameter. Such variation can be useful in optimizing the performance of non-uniform cantilever sensor.

References

[1] Landau, L. D., & Lifshitz, E. M. (1987). Fluid mechanics.

[2] Lighthill, M. J., & Rosenhead, L. (1963). Laminar boundary layers. Clarendon Press, Oxford

[3] Tuck, E. O. (1969). Calculation of unsteady flows due to small motions of cylinders in a viscous fluid. Journal of Engineering Mathematics, 3(1), 29-44.

[4] Sader, J. E. (1998). Frequency response of cantilever beams immersed in viscous fluids with applications to the atomic force microscope. Journal of applied physics, 84(1), 64-76.

[5] Zhang, W., & Turner, K. L. (2006). Fluid Damping and Loading Effects of MEMS Flexural Resonators: Theory and Applications.

[6] Singh, S. S., Pal, P., & Pandey, A. K. (2015). Pull-in analysis of non-uniform microcantilever beams under large deflection. Journal of Applied Physics, 118(20), 204303.

[7] Sagar Singh, S., Pal, P., & Kumar Pandey, A. (2016). Mass sensitivity of nonuniform microcantilever beams. Journal of Vibration and Acoustics, 138(6).

[8] Ashok, A., Kumar, P. M., Singh, S. S., Raju, P., Pal, P., & Pandey, A. K. (2018). Achieving wideband micromechanical system using coupled non-uniform beams array. Sensors and Actuators A: Physical, 273, 12-18.