Modeling of Frequency Locking in a Differential Vibrational Beam Accelerometer

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Summary. Nonlinear dynamic behavior of a generic differential vibrating beam accelerometer is studied numerically and analytically with the emphasize on the frequency locking phenomenon. The device incorporates two tuning fork oscillators attached to a proof mass and described as a weakly nonlinear Kirchhoff beam driven through the self-excitation loop. Our numerical results obtained using the reduced order (RO) Galerkin model indicate that the influence of the geometric nonlinearity and inertial coupling on the locking is minor while the role of the structural coupling is dominant. An analytical prediction of the acceleration range where the locking occurs is obtained by considering two coupled Van der Pol oscillators and is found to be in a good agreement with the numerical results.

Introduction

Microelectromechanical (MEMS) accelerometers are among the most widely implemented and commercially successful micro devices. New challenges imposed by emerging applications continue to stimulate intensive research in the inertial sensors area. Among the approaches allowing performance enhancement are resonant accelerometers, which exploit the dependence of the sensing element natural frequency on the inertial (acceleration) force acting on the proof mass. A large variety of resonant accelerometer designs were reported, including beams directly stretched by an inertial force [1], as well as sensors implementing force amplification or electrostatic softening.

In this work, to explore device's dynamics and modeling approaches, we consider the most generic architecture of a vibrating beam accelerometer (VBA). The device consists of two beam-type tuning fork oscillators attached to a proof mass, Fig. 1(a) [1]. Inertial force acting on the mass results in the increase/decrease of the frequencies of the left (L)/right(R) pairs of the beams $f_{L/R} = f_0\sqrt{1 \pm \gamma Ma/N_E} \approx f_0(1 \pm \gamma Ma/(2N_E))$. Here f_0 is the reference frequency of the unloaded beam, γ is the correction coefficient, M is the proof mass, a is the acceleration and $N_E \gg \gamma Ma$ is the Euler's buckling force. The acceleration is extracted from the measurement of the difference between the two frequencies $\Delta f = (f_L - f_R)$. In the case of nominally identical and uncoupled L and R beams Δf should be zero at $a = 0$. However, as was observed in experiments [2], at small a frequency locking occurs, when two beams oscillate at the same frequency and phase. This phenomenon leads to a "blind zone" in the scale factor curve, limiting the minimal measurable a. As a result, the differential sensing, implemented in these devices to reduce the negative influence of thermal and other environmental effects, cannot be used. Locking (also referred as synchronization) is a well-known phenomenon and the literature on the subject is voluminous [3, 4]. In the MEMS/NEMS area where synchronization can be beneficial, most of the works were focused on the approaches allowing to achieve rather than avoid locking (e.g., see [5] and references therein). However, there is only a limited amount of publications dealing with the modeling of locking in resonant accelerometers [2].

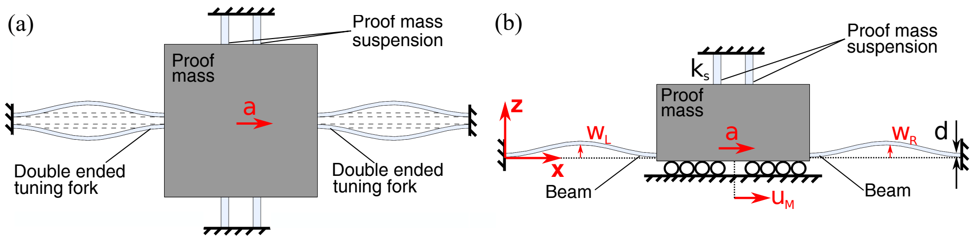

Figure 1: (a) Schematics of a generic vibrational beam accelerometer (VBA). (b) The half-model used in this work.

Model of the device

Due to symmetry only one half of the device is considered, Fig. 1(b). Each of the (assumed to be identical) beams (ties) is clamped at one end and is attached to a proof mass at the other end. The mass moving in the x direction is anchored through the suspension beams. The ties are described in the framework of the nonlinear Kirchhoff beam model, which accounts for a deflection-dependent stretching. By implementing the single-term Galerkin approximation, the beams are described as single DOF oscillators. The dynamics of the device are governed by three coupled nonlinear differential equations in terms of the L and R beams deflections q_L, q_R and the mass displacement u_M

$$\begin{cases} \ddot{q}_L + c\,\dot{q}_L + (1+\sigma u_M)q_L + k_{nl}q_L^3 + \alpha\,(q_L - q_R) = F_L \mathrm{sign}\,(\dot{q}_L) \\ \ddot{u}_M + c_M\dot{u}_M + k_M u_M = a + \mu\left((q_R)^2 - (q_L)^2\right) \\ \ddot{q}_R + c\,\dot{q}_R + (1-\sigma u_M)q_{+}k_{nl}\,(q_R)^3 + \alpha\,(q_R - q_L) = F_R \mathrm{sign}\,(\dot{q}_R) \end{cases} \tag{1}$$

Here c, c_M are the damping coefficients of the beams and proof mass, σ is the inertial force coefficient, k_{nl} is the nonlinear stiffness coefficient, μ is the inertial coupling coefficient [3], k_M is the effective stiffness of the mass suspension and F

is the driving force amplitude. The overdot represents time derivative. The structural coupling coefficient α is extracted from the results of a three-dimensional finite elements modal analysis. Namely, $f_L,\ f_R$ were calculated for the beams with slightly detuned parameters and α was obtained by fitting the veering frequency curves in the vicinity of $a = 0$.

Results

Equations (1) completed by zero initial conditions were solved numerically using Runge-Kutta solver. The beam's frequencies were extracted from the steady-periodic time-history using FFT and zero crossing approaches. The phase between the beams was obtained using Hilbert transform. The results presented here correspond to the following parameters: $c = 2.85 \times 10^{-5},\ c_m = 2.55 \times 10^{-8},\ k_{nl} = 6.75,\ k_M = 0.0115,\ \alpha = 5.75 \times 10^{-5}, \sigma = 2.95 \times 10^{-5}, \mu = 1320, F = 6.68 \times 10^{-8}$. The relative frequency shifts $\Delta f^{rel}_{L/R} = (f_{L/R} - f_0)/f_0$ are shown in Fig. 2(a). In certain range of a the locking occurs and the curves are indistinguishable. The curve representing the maximal phase difference $\Delta\phi$ between the oscillators, Fig. 2(b), indicates that the locking region (in terms of a) is subdivided into two sub-regions - the strong locking where $\Delta\phi$ is close to zero and the weak locking, where a certain phase difference is apparent. Within the locking region the difference in the amplitudes of the two beams (the "mode localization" effect) is more pronounced than outside of the locking area, Fig. 2(c). Our numerical results suggest that for typically small (compared to the beam's width d) values of vibrational amplitudes, the influence of the geometric nonlinearity and of the nonlinear inertial coupling on the locking region is minor. This allows to consider the deflection of the proof mass as quasi-static, and to reduce the problem to two equations of motion which are coupled only through the stiffness coefficients. An additional simplification, which was made due to a periodic character of motion, is to replace the sign function in the driving force such that $\text{sign}(\dot{q}) \approx \arctan(\dot{q}) \approx \dot{q}(1-\dot{q}^2/3) \approx \dot{q}(1-q^2/3)$. As a result, Eqs. (1) are reduced to two coupled Van der Pol equations

$$\begin{cases} \ddot{q}_L + \left[c - F_L\left(1 - q_L^2/3\right)\right]\dot{q}_L + (1 + \tilde{\sigma}\,a)\,q_L + \alpha\,(q_L - q_R) = 0 \\ \ddot{q}_R + \left[c + F_R\left(1 - q_R^2/3\right)\right]\dot{q}_R + (1 - \tilde{\sigma}\,a)\,q_R + \alpha\,(q_R - q_L) = 0 \end{cases} \quad (2)$$

where $\tilde{\sigma}$ is the new acceleration force coefficient. The results based on the asymptotic analytical solution of these equations obtained using averaging [6] are shown in Fig. 2(d). Comparison between the analytically predicted locking region (obtained for the realistic device parameters) and the numerical results showed good agreement between the two. Our modeling approach can be useful also for the analysis of other resonant electrostatically or piezoelectrically actuated accelerometers including tilting devices, devices implementing mechanical force amplification or electrostatic softening.

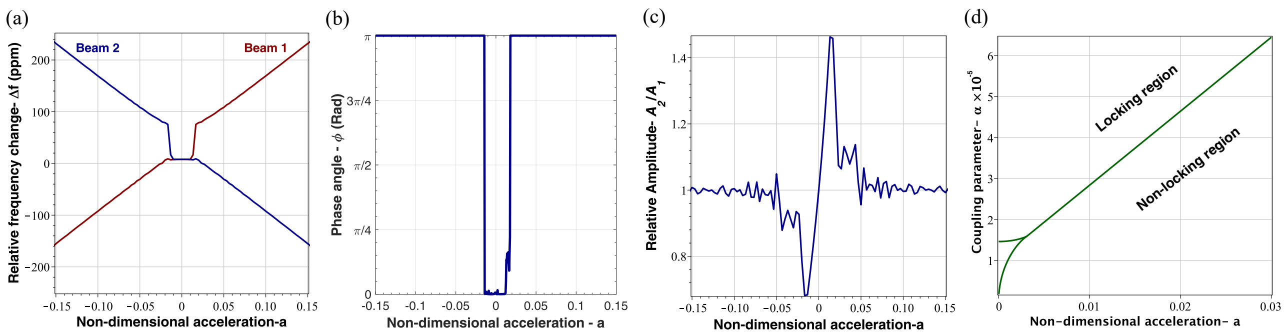

Figure 2: Numerical results obtained by solving Eq. (1) (a) The relative frequency shifts of the each of the beams as a function of the acceleration. In incense of locking the frequency-acceleration dependence curves for each of the beams should cross at $a = 0$. (b) The maximal phase difference between the oscillators as a function of the acceleration. (c) The ratio between the amplitudes of the L and the R beams. (d) Analytical solution - a boundary separating the locked and unlocked regions in the acceleration (detuning)-coupling plane.

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