Measuring nonlinear localisation and isolated curve of solutions in a system of two coupled beams

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Summary. This work presents experimental measurement of nonlinear localisation in a system of two coupled beams. In this system, localisation correspond to vibrations states where the symmetry is broken, and where the energy is mainly localised to one of the two beam only, see e.g. [1]. Using a two degrees of freedom reduced order model, one can show that the localised states arise from a 1:1 internal resonance between two particular linear modes [2]. In the case of forced response, for particular conditions, the localised solution can be depicted under the form of an isolated closed curves (isola) in the amplitude-frequency diagram [3]. Experimental measurements showing nonlinear localisation are presented and compared to the numerical solution, showing good agreement.

Presentation of the system

The system considered here consists of a circular plate that has been machined in order to create to parallel beams as indicated on Fig. 1. The body of the circular plate provides a coupling between the beams, and it also restrains the axial displacement of the beams ends, so that non-linearity occurs due to a coupling between axial and transverse motions.
We consider two particular eigen-modes of the structure depicted on Fig. 1. For those modes, the beams vibrate over a first bending mode shape (in phase or out of phase). The structure has been designed such that both modes interact non-linearly to give a localized mode. Indeed, looking at the mid-beam displacement, the modes can be described with the shape $\phi_1 = (1, 1)$ and $\phi_2 = (1, -1)$. Localisation arise due to a modal interaction of the form $q_1(t)\phi_1 + q_2(t)\phi_2$, where q_1 and q_2 are (in-phase) modal amplitudes, leading to a shape of the form $(a_1 + a_2, a_1 - a_2)$, which eventually tends to the localised shape $(1, \epsilon)$ as the amplitude of the first mode a_1 tend to the amplitude of the second mode a_2.

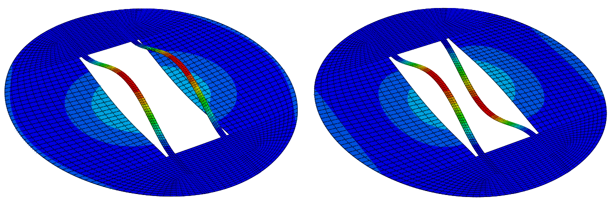

Figure 1: Linear mode shape of interest. Left: in phase mode ($f_1 = 364$ Hz); Right: out-of-phase mode ($f_2 = 367.5$ Hz)

Reduced order model and Numerical solutions

In order to simplify the analysis, a reduced order model is derived using the so called STEP method, particularised to the case of planar structure (see e.g. [4]). Only the two modes of interest are kept in the reduced order model and the reduced set of equation have the following form:

$$\ddot{q}_1 + 2\xi_1\omega_1\dot{q}_1 + \omega_1^2 q_1 + G_1 q_1^3 + C q_1 q_2^2 = f_1 \sin(\Omega t) \tag{1}$$
$$\ddot{q}_2 + 2\xi_2\omega_2\dot{q}_2 + \omega_2^2 q_2 + C q_1^2 q_2 + G_2 q_2^3 = f_2 \sin(\Omega t) \tag{2}$$

where q_i is the modal amplitude of the i-th transverse mode ($i = 1$: in phase mode, $i = 2$ out of phase mode, see Fig.1), ξ_i's are the modal damping ratio, ω_i's are the natural frequencies, f_i's are the the modal forces amplitude and Ω is the excitation frequency. G_1, G_2 and C are the non-linear coefficients obtained through the reduced order model procedure.
Approximated solutions to the system in Eq.(1) are obtained using the Harmonic Balance Method (HBM), coupled with a numerical continuation procedure based on the Asymptotic Numeric Method (ANM) [5].
When the system is forced on the first mode ($f_2 = 0$), if the forcing amplitude is sufficient, it can be observed that the localised solution can be depicted under the form of a closed curve solution in the Amplitude-Frequency diagram (isola), see the left panel of fig.2. When the system is forced on the second mode ($f_1 = 0$), if the amplitude is sufficient, it can be observed that the localised solution stems from the principal resonance curve through a branching point bifurcation, see the right panel of fig.2.

Experimental setup and results

The structure has been machined out of a stainless steel plate using wire cutting. For the experiments, the structure is hanged, which allows to be close to free boundary conditions. The experimental setup is depicted on fig.3.
A magnet and a coil is used to provides excitation to the structure without having an actual physical contact between the structure and the excitation device. To keep the symmetry of the system, another magnet (without coil) is positioned on the opposite side of the plate (see fig.3). The amplitude of the force is controlled by the intensity of the current

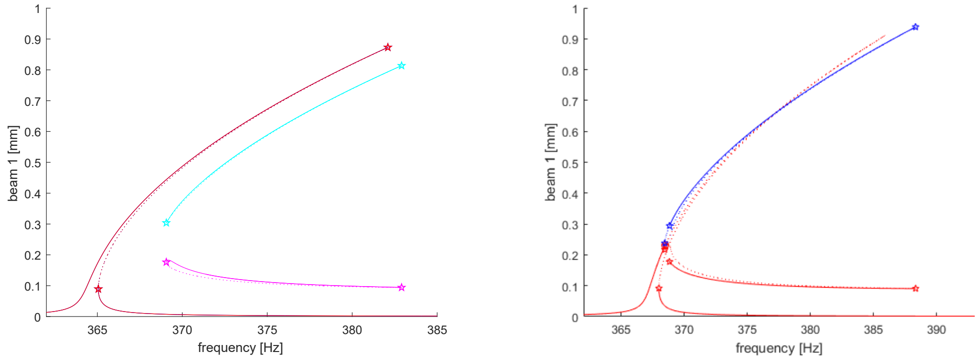

Figure 2: Numerical forced response. Left: force on the first mode, localised states appear as closed curve solution. Right: force on the second mode, localised states arise from the homogeneous (out-of-phase) solution through a branching point bifurcation

sent to the coil, which is monitored using a current clamp. Velocity measurements are carried out using a laser vibrometer.

Example of stepped sine measurements (for a force exciting mainly the second mode) are given on the right panel of fig.3. For low amplitude of forcing, the behaviour of the structure is linear, and both beams vibrates with the same amplitude. When the amplitude increases, the non-linearity starts to be activated and the hardening property of the structure can be observed. Finally, for higher force amplitude, when the displacement amplitude reach a threshold, there is a break of symmetry in the vibration shape of the structure, both beams clearly vibrates with different amplitude. For the highest excitation current (black curve on Fig.3), one can see that the first beam vibrates with an amplitude 7 times greater than the second beam.

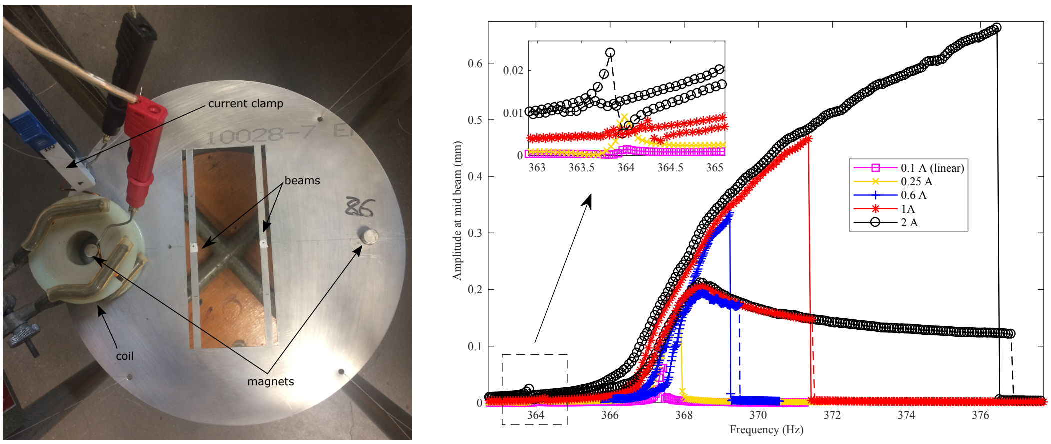

Figure 3: a) Picture of the experimental setup; b) Measurement of localised states when forcing over the second mode (mid-beam amplitude versus frequency of excitation)

Conclusions

This study presents numerical and experimental results about nonlinear localisation in a system of two coupled beams. Experimental measurements using swept sine excitation demonstrate that the localisation can be observed in practice. The numerical and the experimental results agree very well, showing that the reduced order model procedure is able to generate a good representation of the physical system with only a few degrees of freedom.

References

[1] M. King and A.F Vakakis (1995) Mode Localization in a System of Coupled Flexible Beams with Geometric Nonlinearities. *Journal of Applied Mathematics and Mechanics* **75**:127-139.

[2] A.I Manevich and L.I Manevitch, (2005) The Mechanics of Nonlinear Systems with Internal Resonances. *Imperial college press*

[3] A. Grolet, P. Malbos, F. Thouverez (2014)Eigenvalue Method with Symmetry and Vibration Analysis of Cyclic Structures. *Computer Algebra in Scientific Computing. CASC 2014. Lecture Notes in Computer Science* vol 8660. Springer, Cham.

[4] A. Givois, A. Grolet, O. Thomas, and J.F. Deu (2019) On the frequency response computation of geometrically nonlinear flat structures using reduced-order finite element models. *Nonlinear Dynamics* **97**:1747-1781.

[5] B. Cochelin, N. Damil and M. Potier-Ferry (2007) Méthode asymptotique numérique *Hermes Lavoissie*.