Revisiting The Nonlinear Free Vibrations of Hanging Cables – The Use of a Direct Approach on The Partial Differential Equations of Motion

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Summary. In this paper the nonlinear free vibrations of a hanging cable are investigated. The analysis is carried out by applying the method of multiple scales directly on the planar equations of motion for a cable with arbitrary sag and inclination. The only simplification used is that the static strain is small. The methodology allows to obtain results regarding the corrections in modal shape and variations of frequency as function of the initial amplitude. The obtained results show that the hard/softening behaviour that this kind of structure can undergo is in qualitative agreement with previous works.

Introduction

The use of hanging cables as structural elements is commonly found in several engineering application. This type of use can be found in transmission lines, bridges, offshore oil exploration activities, among others. These structures exhibit a rich dynamics, being under extensive efforts in research practice. In order to better understand the dynamic behaviour of these structures, the study of the free vibrations is a problem of interest. This study provides the behaviour of the natural frequencies and modes depending on the structural properties, both from linear and nonlinear perspectives.
The linear modes for a horizontal hanging cable with small sag were deeply investigated in [1], making an analytical study regarding modal properties and the cross-over phenomenon. Following, in [2], the linear modal properties are obtained for inclined taut strings. The results show the mode hybridization and the veering phenomenon that occurs in the inclined configuration. In [3], the linear modes for an arbitrarily sagged inclined cable are obtained. The latter considers an inextensible cable causing modal hybridization to be missed, which can be an important drawback in some problems.
In [4] nonlinear modes are obtained for a vertical beam under tension, with the vertical cable as a particular case. The nonlinear modal properties of a horizontal cable are investigated in [5], using the method of multiple scales (MMS) on a discrete model obtained with the Galerkin method. In [6], the nonlinear modes for an inclined cable with small sag are obtained directly from the partial differential equations (PDEs), disregarding the axial dynamics.
To the best of the authors' knowledge, the investigation of the nonlinear free vibrations of an arbitrary inclined sagged cable with a direct analytical approach on the PDEs is not reported in the literature. One important motivation for this kind of investigation can be found in [7], were it is stated and shown with examples that model discretization using the Galerkin method may furnish wrong qualitative behaviour for the system under study due to obtaining the wrong sign in some coefficients. The major contribution of the present work is then the investigation of nonlinear free vibrations of hanging cables with arbitrary sag and inclination using the MMS directly applied to the PDEs.

Mathematical model

The model herein considered is that of a hanging elastic cable, immersed in fluid, with axial stiffness EA, immersed weight γ_s and axial and transversal masses per unit length m and m_t respectively. The difference between directions is kept in order to allow to include added mass effects. The cable is hanging between two fixed supports with an horizontal distance d and a vertical distance h. It is also considered that the tension $T_e(s)$ and the angle with the horizontal direction $\theta(s)$ in the static configuration are known, with s being the arclength in the static configuration. Being u and v the axial and transversal displacements with respect to the static configuration, the equations of motion obtained from the balance laws for a cable element (see [8] for example) are given as:

$$T_e'(\cos\gamma - 1) - T_e(\theta' + \gamma')\sin\gamma + T_d'\cos\gamma - T_d(\theta' + \gamma')\sin\gamma = m\ddot{u} \tag{1}$$

$$T_e'\sin\gamma - T_e\theta' + T_e(\theta' + \gamma')\cos\gamma + T_d'\sin\gamma + T_d(\theta' + \gamma')\cos\gamma = m_t\ddot{v} \tag{2}$$

In the equations the static equilibrium is already substituted. T_d stands for the dynamic tension variation, γ is the dynamic variation of θ, u is the displacement in the tangential direction and v is the displacement in the transversal direction. Primes denote differentiation with respect to s while dots represent differentiation with respect to time. Now, the hypothesis of small static strain is used, which allows to write for the strain $\varepsilon = \varepsilon_s + \varepsilon_d$, with the subscripts denoting the static and dynamic parts. With that, it is possible to simply write $T_d = EA\varepsilon_d$. Finally, from differential geometry, the compatibility conditions are obtained, namely $\sin\gamma = (u\theta' + v')/(1 + \varepsilon_d)$, $\cos\gamma = (1 + u' - v'\theta')/(1 + \varepsilon_d)$ and $\varepsilon_d = \sqrt{1 + 2\left(u' - v\theta' + uv'\theta' - u'v\theta' + \left(u'^2 + v'^2 + (u\theta')^2 + (v\theta')^2\right)/2\right)} - 1$. The MMS is then applied considering three terms in the expansion for u and v, alongside with three time scales. The development is not reported for the sake of the size of the extended abstract.

Application example

In order to show the functionality of the methodology, some backbone curves for three different cables are obtained. The data for the material properties are obtained from a typical steel catenary riser. The common properties for the three cases are $EA = 2314$MN, $m = 108$kg/m, $m_t = 141.24$kg/m, $\gamma_s = 727$N/m and structural diameter $D = 0.2032$m. For the horizontal cable, $d = 1500$m and $h = 0$m. For the almost vertical cable, $d = 1$m and $h = 1800$m. Finally for the inclined cable, $d = 1500$m and $h = 1800$m. In all the cases the natural length of the cable was taken as the one obtained from an inextensible catenary configuration. The horizontal component of the tension is $T_h = 680550$N for the horizontal and the inclined cable, while is taken as 680N for the vertical one. The backbone curves showing the vibration frequency ω with respect to the natural linear frequency ω_0 as a function of the amplitude A_0 are presented in Fig. 1. It is possible to see

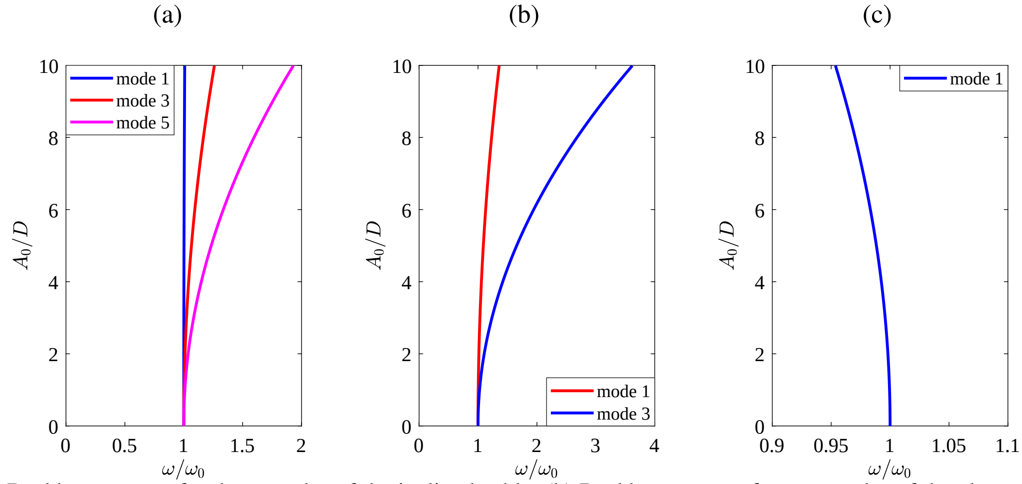

Figure 1: (a) Backbone curves for three modes of the inclined cable; (b) Backbone curves for two modes of the almost vertical cable; (c) Backbone curves for one mode of the horizontal cable. Motion amplitude made dimensionless by the structural diameter.

that both hardening and softening behaviours are obtained, as well as the more intense hardening when going from the chosen inclined model to the almost vertical one. Further examples and analysis of the results are left to the full paper. For the sake of comparison of the qualitative behaviour, the softening behaviour for the horizontal configuration is obtained in [5] while the terms of the fourth order in the expansion are not noticeable. The almost vertical configuration can be compared to the results in [6]. Finally, examples of hardening in inclined cables can be seen in [9].

Conclusions

From the nonlinear equations of motion for a hanging cable, with the small static strain as only simplification hypothesis, nonlinear solutions for the free vibrations were obtained with the method of multiple scales applied directly to the PDEs of motion. The results presented show that the methodology is capable of reproducing some known qualitative results for the dependency of the frequency of vibration with the amplitude of vibration. Further results and comparisons are planned for the full paper and the presentation.

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