Stochastic Dynamics of Inclined Risers Induced by Pulsating Internal Fluid Flow

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Summary. In this work, we numerically explore the stochastic dynamics of inclined marine risers subjected to pulsating internal fluid flow. The presence of geometric nonlinearities with static deflection makes the response of the inclined riser different from conventional top tension risers when subjected to pulsating flows. At first, the riser model is solved via Galerkin method and validated using perturbation approaches. Then, we study the propagation of uncertainties i.e. amplitude and frequency of pulsations in the stochastic model revealing rich and complex dynamics features.

Introduction

Pulsating flow is a phenomenon that affects the oil and gas industries. It occurs due to abrupt perturbations and fluctuations in the internal fluid flow of the riser pipe which in return can affect and influence the vibrational motion of the structure. It occurs due to several reasons such as the nature of the multi-phase flow and sudden geometric changes [1]. Because the value of the excitation amplitude and frequency of fluctuation of the flow are uncertain i.e. stochastic, the influence of the flow can be sever especially if the frequency of these flows are near structural resonances of the riser making them prone to failure by fatigue.

Problem Formulation

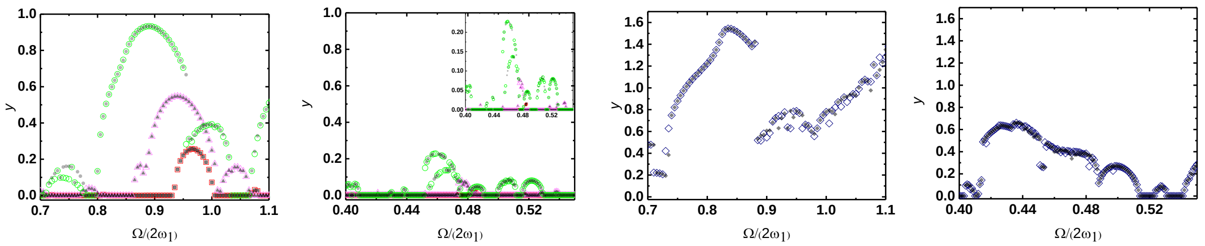
The inclined riser to be analyzed in this work is under mid-plane stretching and subjected to static deflection [2] and pulsating internal fluid flow. Then, the equation that describes the motion of the riser in dimensionless form can be written as

$$\begin{aligned}&\frac{\partial^2 y_d}{\partial t^2}+\frac{\partial^4 y_d}{\partial x^4}+2\sqrt{\beta}v\frac{\partial^2 y_d}{\partial x\partial t}+\sqrt{\beta}\frac{\partial v}{\partial t}(1-x)\frac{\partial^2 y_d}{\partial x^2}+c\frac{\partial y_d}{\partial t}+c_d\frac{\partial y_d}{\partial t}\left|\frac{\partial y_d}{\partial t}\right|+\left(v^2-T+\sigma(1-x)\right)\left(\frac{\partial^2 y_d}{\partial x^2}\right)-\sigma\left(\frac{\partial y_d}{\partial x}\right)\\&-\left(\eta\left(\int_0^1\left(\frac{dy_s}{dx}\right)^2dx+\int_0^1\left(\frac{\partial y_d}{\partial x}\right)^2dx+\int_0^1 2\frac{dy_s}{dx}\frac{\partial y_d}{\partial x}dx\right)\right)\left(\frac{\partial^2 y_d}{\partial x^2}\right)-\eta\left(\int_0^1 2\frac{dy_s}{dx}\frac{\partial y_d}{\partial x}dx+\int_0^1\left(\frac{\partial y_d}{\partial x}\right)^2dx\right)\left(\frac{d^2 y_s}{dx^2}\right)=0\end{aligned} \tag{1}$$

where y_d is the deflection of the riser along position x and time t, β is fluid mass parameter, σ is self-weight parameter, v is internal fluid flow velocity, η is nonlinear geometric parameter, c is structural damping, c_d is external fluid damping and T is the applied top tension. Due to flow fluctuations, the internal velocity is assumed to have the form $v=V\left(1+\gamma\cos(\Omega t)\right)$ where V is the magnitude of the internal velocity, γ is a detuning parameter between 0 and 1 and Ω is the excitation frequency. Equation (1) is solved via Galerkin method utilizing the procedures prescribed in [3] and [4] and validated using perturbation method defined in [5-7]. Next, we consider a probabilistic frame of work in which the amplitude γ and the excitation frequency Ω are random variables. Then, we study the influence of type of different probability distributions (PDF) on the dynamic response of the structure using Monte Carlo (MC) simulations with 2^{13} samples.

Numerical Results

At first, the deterministic model is analyzed. Due to the quadratic nature of the internal fluid flow, the excitation frequency is expected to occur at Ω and 2Ω because of the nature of the parametric excitation. The dynamic response of the lowest three modes is depicted in Fig.1

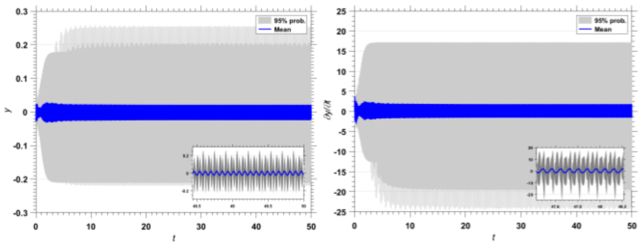


(a) (b) (c) (d)

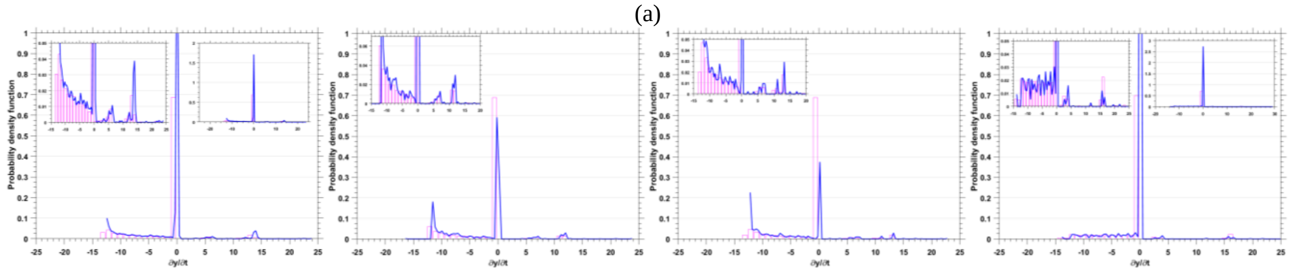
Figure 1: Multi-modal Frequency response curves around the first mode of vibrations for $V=\sqrt{\sigma}$, $c_d=0.416$ at x=0.48. (a,b) Backward Sweep (□) $\gamma=0.25$, (△) $\gamma=0.50$, (○) $\gamma=0.75$. The inset is magnified results for case (b). (c,d) (◇) $\gamma=1$. Filled shapes denotes forward sweep.

We observe, in Fig. 1, the influence of the different components that exists in the system as a result of the interaction. The influence of the softening nonlinearity is less apparent due to the competing effects between the first mode and contributions from other modes that exist in the response. This is observed very well in Fig. 1b in comparison to other cases. In addition, the interaction of other resonances with the response of the solution near Ω and 2Ω become more visible at higher fluctuating velocity as the solution demonstrate chaotic behavior.

Next, we consider the stochastic response of the riser. The results from the MC method are divided into ten categories considering the two excitation frequency ranges and the combination of the distributions for the amplitude utilizing Beta and uniform distributions where they are applicable. As an example, we demonstrate the response of the riser under Beta distribution for excitation at a frequency of $\Omega/2\omega_1 = 0.464$ in Fig.2



(a)



$t = 19.99$ $t = 29.99$ $t = 39.99$ $t = 50$

(b)

Figure 2: $\Omega/2\omega_1 = 0.464$ (a) Illustration of the nominal value (Blue line) and the 95 % envelop (grey shadow) of the displacement (left) and the velocity (right) of the riser. The box in each figure is a magnified response of the riser. (b) Evolution of the normalized probability density function as a function of the rise velocity at different times. The box in each figure is a magnified distribution of the probability density function.

The response of the riser displacement is exhibited on the left Fig. 2a and we plot the velocity in order to have a better representation of the riser response. The distribution influences type of response because at this frequency the riser excite not only secondary terms due to quadratic nonlinearity but also primary due the squaring of terms. Due to this fact, the mean value is observed to be superseded by different excitation amplitudes constituting the envelope. The propagation of the probability density function at different time interval demonstrate that it is stationary. The main feature is attributed to the primary excitation of the riser structure.

Conclusions

In this work, the stochastic dynamics of inclined risers is studied considering the influence of pulsating internal fluid flow. The presence of static deflection under the influence of geometric nonlinearity causes multiple resonances to exist. This influences the stochastic response of the riser examined under different distributions. As a result, the response of the riser in the Monte Carlo simulations revealed interesting complex and rich dynamic features

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