The effect of the fibre orientation on the geometrically non-linear vibrations of tow placed composite plates with real clamped boundaries

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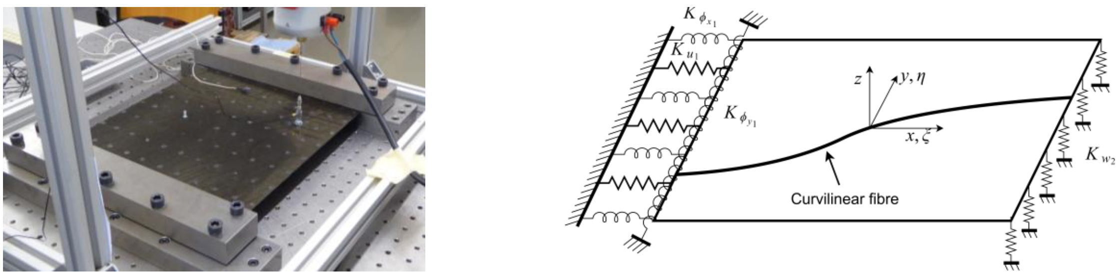
Summary. Curvilinear reinforcement fibres can, within limits, be used to tailor the stiffness of composite laminated plates so that they have specific dynamic properties. The dynamic response of a plate to external forces is strongly affected both by boundary conditions and by the stiffness of the plate near the boundaries. Using a combination of experimental and numerical analyses, this work intends to analyse how the stiffness of the boundaries and the stiffness of tow placed composite plates near the boundaries interact to influence the geometrically non-linear response to harmonic external forces.

Introduction

The application of curvilinear reinforcement fibres in laminated composite plates leads to a variation of the stiffness within the plate domain, allowing designers to tailor the plate so that it meets specific demands more efficiently. Hence, there has been a considerable amount of research on this type of laminated plates [1]. The stiffness of the plate near the boundaries and its relation with the boundary conditions themselves have a major influence on the dynamic response. Classical boundary conditions, i.e., combinations of free, hinged or clamped edges, are typically considered by researchers. However, in practice it is not possible to exactly implement classical boundary conditions, with the clamped edge case particularly difficult to approach [2]. A more realistic representation of "real clamps" is achieved by considering elastically restrained edges [3].
This work intends to analyse how the non-linear response is affected by changes of the fibre path near the plate's edges, when the latter are elastically restrained. For that purpose, a physical/mathematical model, which considers that the edges are supported on translational and rotational springs, is developed. The values of the stiffness of the boundary springs are adjusted so that the theoretical linear modes of vibration approach experimentally identified ones [4].

Mathematical model

An equivalent single-layer type formulation, based on Kirchhoff's hypothesis, but accounting for geometrical non-linear terms in the Von Kármán sense, is adopted. Components *u*, *v* and *w* (respectively in directions *x*, *y* and *z*) of the middle-plane displacements are written as series with products of shape functions by generalized coordinates and the principle of virtual work is applied. Apart from new stiffness terms due to the elastically restrained edges, the equations of motion are similar to the ones given in [5].



(a) (b)

Figure 1: Experimental set-up (a) and schematic representation of a plate supported by distributed springs (b); only some springs are depicted.

In the laboratory, the plate is fixed only on two opposite edges using stiff steel blocks, as shown in Figure 1 (a). The corresponding model with translational and torsional distributed springs is shown in Figure 1 (b), where some distributed springs are drawn on the left-hand side and one on the right-hand side, for the sake of clarity. K_{u_i} and K_{w_i} represent the values of the stiffness per unit length of translational springs along the *x* and *z* axes, respectively; $K_{\phi_{x_i}}$ and $K_{\phi_{y_i}}$ represent the torsional stiffness per unit length of springs that rotate, respectively, about the *x* and the *y* axes. Index *i* is equal to 1 if ξ=-1 and equal to 2 if ξ=1. Displacement at the boundaries is small, so it is still reasonable to assume that these stiffness coefficients are constant, even though this work is in the geometrically non-linear regime.

The virtual works done by the distributed forces and moments due to the various boundary springs are

$$\delta W_{K_u} = -\int_{-1}^{1} K_{u_1} u(-1,\eta,t)\,\delta u(-1,\eta)\frac{b}{2}\mathrm{d}\eta - \int_{-1}^{1} K_{u_2} u(1,\eta,t)\,\delta u(1,\eta)\frac{b}{2}\mathrm{d}\eta \quad (1)$$

$$\delta W_{K_w} = -\int_{-1}^{1} K_{w_1} w(-1,\eta,t)\,\delta w(-1,\eta)\frac{b}{2}\mathrm{d}\eta - \int_{-1}^{1} K_{w_2} w(1,\eta,t)\,\delta w(1,\eta)\frac{b}{2}\mathrm{d}\eta \quad (2)$$

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$$\delta W_{K_{\phi_y}} = -\int_{-1}^{1} K_{\phi_{y1}} w_{,x}(-1,\eta,t)\,\delta w_{,x}(-1,\eta)\frac{b}{2}\mathrm{d}\eta - \int_{-1}^{1} K_{\phi_{y2}} w_{,x}(1,\eta,t)\,\delta w_{,x}(1,\eta)\frac{b}{2}\mathrm{d}\eta \tag{3}$$

$$\delta W_{K_{\phi_y}} = -\int_{-1}^{1} K_{\phi_{x1}} w_{,y}(-1,\eta,t)\,\delta w_{,y}(-1,\eta)\frac{b}{2}\mathrm{d}\eta - \int_{-1}^{1} K_{\phi_{x2}} w_{,y}(1,\eta,t)\,\delta w_{,y}(1,\eta)\frac{b}{2}\mathrm{d}\eta \tag{4}$$

The ordinary differential equations of motion are obtained by applying the principle of virtual work and have the following form [5]

$$\mathbf{M}\ddot{\mathbf{q}}_w(t)+\mathbf{C}\dot{\mathbf{q}}_w(t)+\mathbf{K}\mathbf{q}_w(t)+\mathbf{K}_{nl}\left(\mathbf{q}_w(t)\right)\mathbf{q}_w(t)=\mathbf{p}_w(t) \tag{5}$$

where **M** is the mass matrix, **C** a matrix due to viscous damping, **K** a constant stiffness matrix and $\mathbf{K}_{n\lambda}(\mathbf{q}_w(t))$ a stiffness matrix that represents non-linear effects. Most of the terms due to the elastic supports are explicitly given in [4], they alter stiffness matrix **K**; the remaining matrices are given in [5]. Vector $\mathbf{q}_w(t)$ contains the transverse generalised coordinates; $\mathbf{p}_w(t)$ is the vector of generalised transverse external forces. Although we are accounting for in-plane displacements at the boundaries, these are still extremely small and so is the in-plane inertia. Hence, the latter is neglected and, consequently, the non-linearities of the set of equations of motion (5) are of the cubic type.

Sample results and closure

Several frequency response functions were measured (Figure 2) and employed to obtain natural modes of vibration in the linear regime [4]. With this data, the stiffnesses of the boundary springs were adjusted, resulting in models that provide modal data that is reasonably similar to the experimentally obtained results [4]. As an example, Table 1 shows that the linear natural frequencies of vibration computed by the model with elastic supports ($\omega_i^{Springs}$) are much closer to the experimental ones ($\omega_i^{experimental}$) than the frequencies computed using a model that adopts theoretical, exact, clamped boundary conditions (ω_i^{CFCF}).

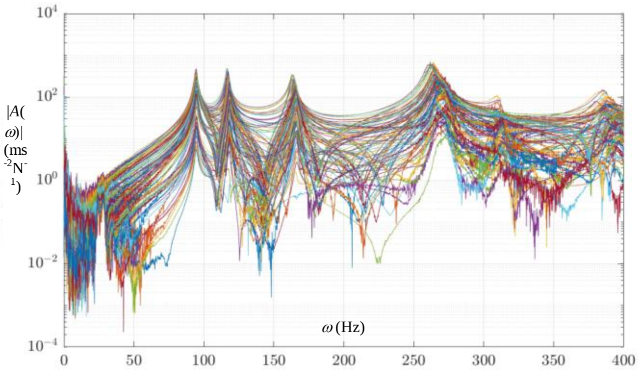

Figure 2: Frequency response functions from measurements on the plate represented on Figure 1 (a).

Table 1. Experimental and theoretical natural frequencies (in Hz).

Mode number	1	2	3	4	5	6	7
$\omega_i^{experimental}$	95.13	117.6	165.0	258.0	275.8	307.3	361.6
$\omega_i^{Springs}$	97.35	116.1	175.7	277.5	286.3	321.6	405.4
ω_i^{CFCF}	136.7	153.1	211.7	315.7	379.9	403.0	486.1

Harmonic excitations are applied and the non-linear response is computed using the harmonic balance and continuation methods [5]. Simulations are carried out for diverse fibre paths. The dynamic response is analysed, with particular attention paid to the combined influence of the elasticity of the supports and of the plate’s stiffness near the boundaries.

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