Modelling and dynamics of smart composite box beam with nonlinear constitutive behaviour of active elements

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Summary. The presented research discusses the mathematical modelling of composite thin-walled beams with embedded active elements made of piezoceramic materials. The the analysis the nonlinear relations in piezoceramics with respect to electric field (electrostrictive effect) is considered to properly capture the behaviour of the transducer in near-resonant conditions. Moreover, the importance of disregarding/accounting for the magnetic effects is highlighted. In the adopted model of the piezoceramic actuation the bending load to the hosting composite beam is achieved by means of d_{31} piezoelectric effect. The governing equations of the system are derived using Hamilton's principle for a beam undergoing complex deformation involving transverse and in-plane shearable bending, torsion and axial deformations. The obtained system of partial differential equations is transformed into a set of ordinary ones by the Galerkin discretization method. The results of performed numerical studies show the importance of non-linear terms for accurate prediction of systems dynamic properties; in particular one can observe the softening phenomenon near the resonance zone due to the nonlinear characteristics of the PZT layers and electrostrictive effect.

Over recent 10-20 years smart composite elements have received a considerable attention due to their potential for designing adaptive structures that are both light in weight and possess adaptive capabilities. Due to their unique properties like e.g. high strength-to-weight ratio and high structural damping the composite based smart structures are very competitive for many designs. In particular active transducers based on piezoelectric/piezoceramic materials have found numerous applications ranging from astronautics and aerospace technology to the automotive industry and civil engineering. Typical examples might be spacecraft antennas, helicopter rotor blades, wind turbines, multi-stable morphing shells, bridge elements etc.

Modelling of smart composite structures

The physics involved in piezoelectric theory can be regarded as a coupling between Maxwell's equations of electromagnetism and elastic stress equations of motion. The coupling takes place through the piezoelectric constitutive equations providing the relationships between the tensors of stress and strain and the vectors of electric field and electric flux density.

The system of Maxwell's equations in vector form is written as

$$\begin{aligned} \nabla \cdot \mathbf{D} &= \rho_e \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= \dot{\mathbf{B}} \\ \tfrac{1}{\mu}(\nabla \times \mathbf{B}) &= i_b + \dot{\mathbf{D}} \end{aligned} \tag{1}$$

where $\mathbf{D}$ is the electric flux density vector, also known as the electric displacement vector, ρ_e is the charge density, $\mathbf{B}$ is the magnetic induction, $\mathbf{E}$ is the electric field intensity vector and μ is the magnetic permeability constant. The over dots represent differentiation with time t.

In the literature there are mainly three approaches to deal with electromagnetic effects when modelling the behaviour of piezoelectric domain [1]:

- electrostatic approach when all magnetic effects ale completely ignored. This involves $\mathbf{B} = \dot{\mathbf{D}} = \rho_e = i_b = 0$. Therefore the system of Maxwell's equations is reduced to $\nabla \cdot \mathbf{D} = 0$ and $\nabla \times \mathbf{E} = 0 \Rightarrow \mathbf{E} = -\nabla\phi$. This approach is the simplest one and simultaneously the most common one even when studying dynamics of smart structures. This is due to the fact that the electric potential ϕ is directly available,
- quasi-static approach that rules out some but not all the magnetic effects. Typically $\rho_e = i_b = 0$ but D and B are time-dependent. Thus, the $(1)_2$ implies that there exists a magnetic potential vector $\mathbf{A}$ such that $B = \nabla \times \mathbf{A}$. Therefore, the electric potential is related to electric field and changes in magnetic potential: $\mathbf{E} = -\nabla\phi - \dot{\mathbf{A}}$,
- fully dynamic when $\mathbf{A}$ and $\dot{\mathbf{A}}$ are left in the analysis. Depending on the type of material, body charge density ρ_e and body current density i_b can also be non-zero.

The well known physically linear relationships between the tensors of stress and strain and the vectors of electric field and electric displacement for piezoelectric materials are applicable to a particular case where nonlinear effects are negligible [2, 3]. However, numerous theoretical and experimental studies suggest the piezoceramics can exhibit nonlinear constitutive properties resulting from high electric fields, near resonant operation regimes or stress-strain hysteresis [4, 5, 6, 7, 8].

To properly capture these phenomena authors postulate to enhance the classical piezoelectric constitutive formulation by adopting the higher-order relations with respect to electric field [9]

$$\begin{aligned} \boldsymbol{\sigma} &= \mathbf{C}\boldsymbol{\varepsilon} - \mathbf{e}\mathbf{E} - \hat{\mathbf{b}}\ \mathrm{sgn}(E_3)\mathbf{E}^2 \\ \mathbf{D} &= \mathbf{e}\boldsymbol{\varepsilon} + \boldsymbol{\xi}\mathbf{E} + \boldsymbol{\chi}\mathrm{sgn}(E_3)\mathbf{E}^2 \end{aligned} \tag{2}$$

In the above relations $\mathbf{C}$ stands for the second order piezoceramic elasticity tensor at constant electric field, $\mathbf{e}$ is the tensor of piezoelectric coefficients, $\boldsymbol{\xi}$ is second order permittivity tensor, $\hat{\mathbf{b}}$ is effective electrostrictive constants tensor, $\boldsymbol{\chi}$ is third order electric susceptibility tensor. Moreover, the variables $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ stand for stress and strain tensors, respectively.

The submitted research is a continuation of former author's studies on dynamics of smart composite thin-walled beams [10, 11]. In the former one an electromechanical coupled theory is used to develop the equations of motion of a rotating thin-walled beam with surface bonded/embedded piezoelectric transducers. In the mathematical model of the hybrid structure, the non-classical effects like material anisotropy, rotary inertia and transverse shear deformation as well as an arbitrary beam pitch angle and hub mass moment of inertia are incorporated. It has been shown this approach results in an additional equation of motion for the hub sub-system and significantly enhances the generality of the formulation. Comparing to the purely mechanical model with simplified approach, the proposed electromechanical one introduces additional stiffness-type couplings between individual degrees of freedom of the system. In the following paper [11] the dynamics of layered composite piezo-beam with lamination scheme exhibiting the circumferentially uniform stiffness properties of the cross section has been presented. Moreover, the two-way coupling interaction involving the spatial distribution of electric field in the piezoceramic domain has been discussed.

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