The effect of temperature on thermo-elastic plate response: FE and reduced model

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Summary. In the present work a thermo-elastic model of a circular plate is analyzed. Nonlinear oscillations of a heated plate subjected to dynamic loading are studied. A model of the plate is based on the geometrically nonlinear Mindlin plate theory. Two different approaches are used to study the problem: a pure numerical study by the finite element method (FEM) and an analytical study based on the harmonic balance method applied to the reduced model taking into account the first vibration mode. The influence of the loading and the elevated temperature on dynamic behaviour is studied for buckling and post buckling states.

Introduction

The plates are used in many technological areas. Being fundamental structural elements often the plates are subjected to mechanical and thermal loadings which lead to intensive large amplitude vibrations. The temperature can change the reaction of mechanical structures and in some cases, close to critical points, even a small temperature variation may give unexpected change in the system response. The case when the studied problem of thermally induced vibrations or vibrations produced by coupled mechanical and thermal loads is nonlinear its detailed investigation is specially important. Detailed analysis of local and global nonlinear dynamics of plates for various thermal and mechanical loads are conducted in [1,2].

In this work we analyze the effect of thermal loading on the nonlinear vibrations of a circular plate according to the extended Mindlin plate theory taking into account the geometrically nonlinearities due to the large displacements. FEM is used to study the response of the plate in the time domain. A reduced model based on the Galerkin ortogonalization method is the second approach to study the problem. By harmonic balance method the response of the plate is studied in the frequency domain. The resonance curves are determined from the analytically obtained modulation equations. Stability of the solution is studied in details.

Physical method

A circular plate with radius *R* and thickness *h* vibrating asymmetrically due to a harmonic mechanical loading is taken into consideration. It is accepted that the plate gets an elevated temperature instantly and the temperature is distributed uniformly along the plate surface and thickness. Based on extended Mindlin plate theory, and accepting that the inertia term in mid-plane can be neglected, the equations of the plate vibration are:

$$Ah\left[\frac{\partial^2 u}{\partial r^2}+\frac{\partial w}{\partial r}\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}+\frac{1}{2r}(1-\nu)\left(\frac{\partial w}{\partial r}\right)^2\right]=0$$

$$D\left[\frac{\partial^2\psi}{\partial r^2}+\frac{1}{r}\frac{\partial\psi}{\partial r}-\frac{\psi}{r^2}\right]-k^2Gh\left(\frac{\partial w}{\partial r}+\psi\right)-c_2\frac{\partial\psi}{\partial t}-\frac{\rho h^3}{12}\frac{\partial^2\psi}{\partial t^2}=0$$

$$k^2Gh\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial\psi}{\partial r}+\frac{\psi}{r}\right)+Ah\left[\frac{\partial u}{\partial r}+\frac{1}{2}\left(\frac{\partial w}{\partial r}\right)^2+\frac{\nu}{r}u-\frac{1+\nu}{h}\alpha_T\Delta T\right]\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)+$$

$$Ah\left[\frac{\partial^2 u}{\partial r^2}+\frac{\nu}{r}\frac{\partial u}{\partial r}-\frac{\nu}{r^2}u+\frac{\partial w}{\partial r}\frac{\partial^2 w}{\partial r^2}\right]\cdot\left(\frac{\partial w}{\partial r}\right)+c_1\frac{\partial w}{\partial t}-\rho h\frac{\partial^2 w}{\partial t^2}=-p(r,t)$$

Where *u* and *w* are in-plane and transversal displacements, ψ is the angular rotation, c_1 and c_2 are damping coefficients, ρ is the density, υ is Poison's ratio, E is the Young modulus, T is the temperature and

$A=\dfrac{E}{1-\nu^2}$, $D=Ah^3/12$

The boundary conditions for a clamped, in-plane fixed plate are:

$$u(0,t)=u(R,t)=w(R,t)=0,\psi(R,t)=0$$

The model of the plate represented by PDE is reduced to the ODE by the Galerkin method based on the modes projection and taking into account excitation, distributed according to the first mode shape. For the first mode reduction we obtain just one nonlinear differential equation:

$$\ddot{q}_1+2\xi_1\omega_1\dot{q}_1+\omega_1^2q_1+F_{NL1}q_1^3+F_{NT1}\Delta Tq_1=F_{NP1}\sin\omega t$$

Here F_{NL1}, F_{NT1} and F_{NP1} are constants obtained numerically, ω_1 is the first natural frequency and ξ_1 is a damping coefficient. Applying harmonic balance method, after some algebraic manipulations we obtained a cubic algebraic equation:

$$-16P^2+9z^3\gamma^2+z^2(24\lambda T\gamma+24\alpha\gamma-24\gamma\omega^2)+z(16\lambda^2T^2+32\lambda T\alpha+16\alpha^2$$
$$-32\lambda T\omega^2-32\alpha\omega^2+16\beta^2\omega^2+16\omega^4)=0$$

which is solved analytically.

FEM and numerical results

Using the FE program ANSYS Mechanical APDL the plate is discretized by a quad mapped mesh with 2700 elements and 2884 nodes. Four nodes finite element SHELL 181 is used. The response of the plate is studied for different values of the loading, excitation frequencies and different temperatures. Generally, the elevated temperatures lead to enlarging of the amplitude of vibrations. At Fig. 1 we show time history diagrams of response of the plate centre for 4 different temperatures. It is clear seen that at ΔT=30 and ΔT=40 the plate buckles and continues to vibrate around a new equilibrium state.

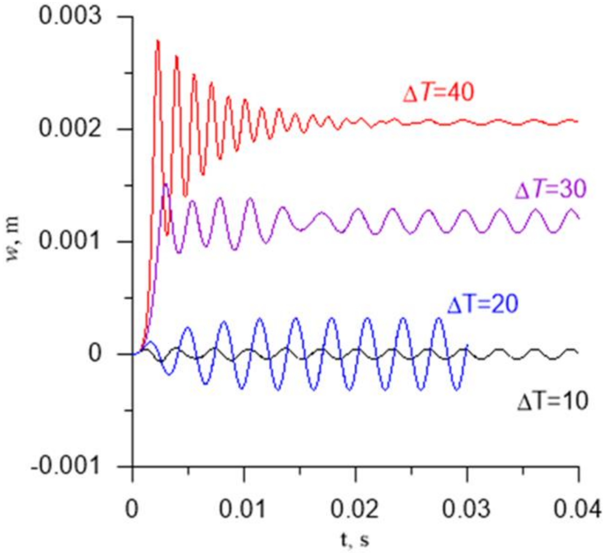

Fig. 1: Time history diagrams of the plate centre of the plate subjected to harmonic loading with p=1500 N/m^2 and ωe=312.5 Hz.
Black colour – ΔT=10; Blue colour – ΔT= 20;
Purple colour – ΔT= 30 , Red colour – ΔT=40.

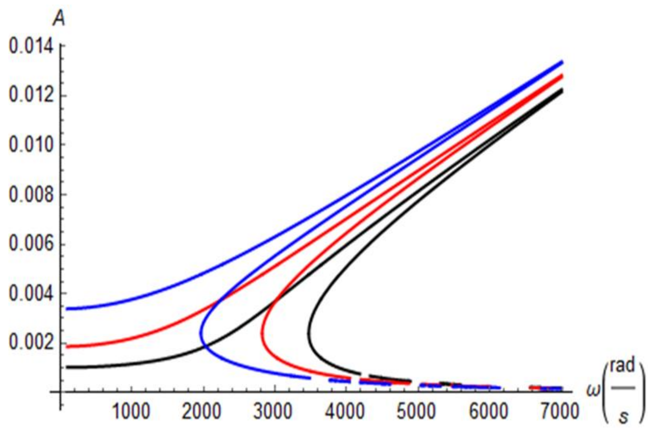

Fig. 2: Resonance curves by HBM for ΔT = 20 (black), ΔT = 30 (red) and ΔT = 40 (blue).

HBM solution

The algebraic equation obtained by HBM is solved for different values of the loading parameters and the temperature. An example for such solutions are the resonance curves shown in Fig. 2. The hardening behaviour of the resonance curves are clearly expressed due to the large deflections. The elevated temperature increases the amplitudes and changes the resonance curve in the direction of lower frequencies. It is seen that multiple and unstable solutions can occur.

Conclusions

Geometrically nonlinear thermo-elastic vibration of a Mindlin circular plate is studied by two different methods. In the first approach by FEM is demonstrated that the elevated temperature can change dramatically the response of the plate and could provoke the plate to complex response, including buckling and bifurcations. The second approach allows to obtain easily the resonance curves and to study the influence of the loading parameters and the elevated temperature on the behaviour of the plate. The computations based on the first mode reduction show the stiffening effect of the resonance curve.

References

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