Dissipation Effects in Mechanical Systems

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Summary. Dissipation is part of every physical system. This paper looks at a possible classification of such effects based on the dynamic response characteristics. It highlights some interesting non-trivial characteristics that particularly enter when a system otherwise unstable is stabilized by dissipative effects. These response properties can have various useful applications, and they can contribute to determining the dominant dissipation sources in mechatronic systems.

Introduction

Mechanical system models generally include forces that arise from fundamental physical phenomena [2]. Dissipative effects represent one of such basic classes of forces. The dissipative mechanisms at the micro-scale usually lead to some resultant effective representations at the macro-scale, which are commonly used in dynamic system models. These generally appear in two fundamental forms: one is when the damping/dissipative force model is directly proportional with the velocity, i.e., viscous and structural damping models, and the second group is when the dissipative force model depends on the sign of the velocity but not necessarily the magnitude, i.e., friction models [3].

Dynamic model

The mathematical models for these can be written considering one degree-of-freedom canonical models as

$$m\ddot{q} + b\dot{q} + kq + c\,\mathrm{sgn}\,(\dot{q}) = f, \tag{1}$$

where q represents the generalized coordinate parametrizing the single degree-of-freedom as a function of time, m is the related to the generalized mass/inertia, k is the stiffness, and b is the viscous/structural damping coefficient. The magnitude of the friction force is denoted by c, and f represents the other forces acting on the system.

If mechanical systems without dissipation are considered then one possible classification can be based on their stability. The behaviour of a mechanical system model with purely inertia and stiffness terms is always stable by the Lyapunov sense. These inertia and stiffness terms represent the physical system elements and generally derive from the Lagrangian of the system. However, this usually represent only one part of the physical system.

The other forces that can enter in f determine in general whether the overall system is stable or not. Instability can result from various effects that can be introduced by these forces. Several of the major effects include some form of time delay. One important group of applications is virtual environments where f include discrete-time computer generated representation of virtual interaction forces.

Based on Eq. (1), the two limit cases are considered where either only viscous/structural damping, $b\dot{q}(t)$ term, or friction-based dissipation, $c\,\mathrm{sgn}\,(\dot{q}(t))$ term, exists. As an illustration, these are shown for the case where f vanishes and when f represents a virtual stiffness with formula $f(t) = -k_{\mathrm{p}}q(t_j)$, where k_{p} is the virtual stiffness coefficient, and $q(t_j)$ represents the sampled position in every jth sampling instant, i.e., $t_j = jt_{\mathrm{s}}$ with $j \in \mathbb{Z}$, and t_{s} denotes the sampling time. With the reduced number of free parameters, Eq. (1) is rewritten

$$\ddot{q}(t) + 2\zeta\omega_{\mathrm{n}}\dot{q}(t) + \omega_{\mathrm{n}}^2 q(t) + \sigma\omega_{\mathrm{n}}^2\,\mathrm{sgn}\,(\dot{q}(t)) = -\tfrac{k_{\mathrm{p}}}{m}q(t_j), \quad t \in [t_j, t_j + t_{\mathrm{s}}), \tag{2}$$

where the natural angular frequency $\omega_{\mathrm{n}} = \sqrt{k/m}$, the damping ratio $\zeta = b/(2m\omega_{\mathrm{n}})$, and $\sigma = c/k$. In order to obtain a more compact model, the dimensionless time $T = \omega_{\mathrm{n}}t$ is also introduced. Thus, the dimensionless sampling instant is $T_j = j\omega_{\mathrm{n}}t_{\mathrm{s}} = j\tau$. Based on these, the equation of motion can be given as

$$q''(T) + 2\zeta q'(T) + q(T) + \sigma\,\mathrm{sgn}\,(q'(T)) = -p\,q(T_j), \quad T \in [T_j, T_j + \tau), \tag{3}$$

where $p = k_{\mathrm{p}}/k$, and prime denotes the differentiation with respect to the dimensionless time.

Dissipation analysis

The first case of f results in a stable system, while the second case produces a system that can be unstable without dissipation, and adding dissipative effects can stabilize the system. In the case of only viscous/structural damping the response properties are well-known; it generally shows exponential decay type characteristics. In the case of unforced and in the case of applied virtual stiffness, the vibration characteristics are presented in Fig. 1a and Fig. 1b, respectively.

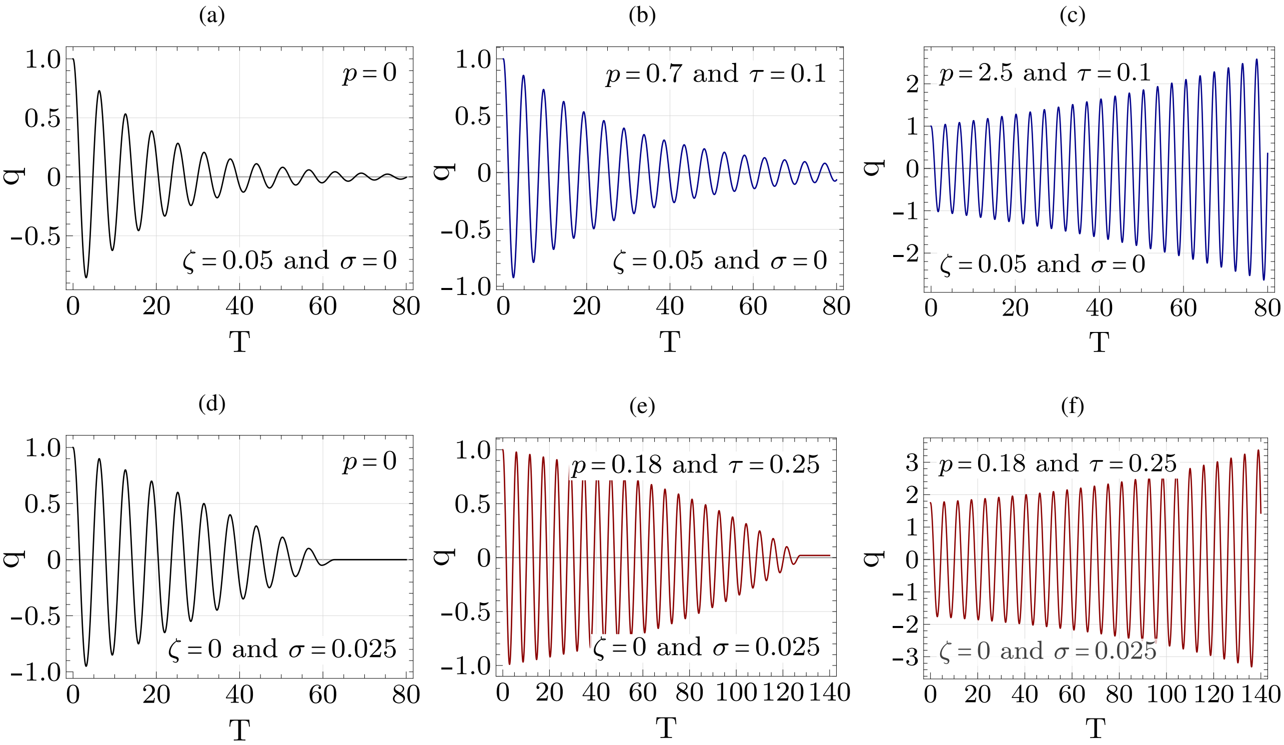

Figure 1: Fundamental vibration shapes due to the effect of different kind of dissipation phenomena

It can be also observed that the effect of sampling results in an effective negative viscous damping; thus with increasing virtual stiffness coefficient the effect of dissipation decreases, and there may be loss of stability, as it is shown in Fig. 1c. However, for the second case where only friction-based dissipative effects exist, the response characteristics are much less trivial.

There is a difference between the behaviours of a stable system and a dissipation-stabilized system. In the case when f vanishes, linear decaying amplitude exists, while in case of the additional virtual stiffness, the so-called concave amplitude decay [1] can be observed. These behaviours are shown in Fig. 1d and Fig. 1e, respectively. Due to the presence of friction-based dissipation, the system become sensitive to the initial conditions resulted in an unstable limit cycle. The resulted in unstable motion is presented in Fig. 1f.

It is noted that the behaviours of such concave decaying characteristics in various systems when generating discrete-time virtual interaction forces were experimentally observed [1]. Such observations can generally indicate that Coulomb friction may act as dominant dissipative and stabilizing effect for these cases.

Conclusions

In this paper, the vibration decay characteristics of mechanical systems were investigated by considering either viscous/structural damping or dry friction as the primary source of dissipation. The fundamental vibration shapes were presented for the unforced case, and for that case when the acting force represents virtual stiffness. It was highlighted that the influence of different physical dissipation resulted in some non-trivial vibration decaying characteristics that particularly enter when a system otherwise unstable is stabilized by dissipative effects.

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