Strongly Nonlinear Forced Damped Model for the Dynamics of the Valve Spring

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Summary. The preloaded valve spring of an internal combustion engine is mathematically modeled as a one-dimensional finite non-homogenous forced-damped chain using the Hertzian contact law, and its strongly nonlinear dynamics is analytically studied. An approximated analytical single-frequency solution in the form of time-periodic and spatially localized state is derived. The theoretical results are confirmed by numerical simulations as well as experimental test results. This indicates that the strongly nonlinear chain allows a single-frequency approximate solution.

Introduction

The interplay between nonlinearity and discreteness supports time-periodic and spatially localized solutions, which are commonly referred to as discrete breathers (DBs) [1]. Exact solutions for symmetric discrete breathers were derived for the Hamiltonian model [2] and for the case of a homogenous external forcing with restitution coefficient less than unity [3]. Recently, we developed a discrete nonhomogeneous model for the nonlinear dynamics of the valve spring [4]. This model revealed two qualitatively different states of the periodic responses; we referred to them as propagating states and edge states. The propagating states are characterized by weak localization, and the edge states—by strong localization at the forced edge. To meet more realistic conditions of the valve train, in this study, we extend our previous model in two ways. First, using the Hertzian contact law to describe all the valve-valve seat interactions in each period of excitation. Second, we include the preload effect. This implies that in our model, both the precompression and the non-smooth behavior caused by the separation exist.

Model description and analytical treatment

Following our previous paper [4], the valve spring is mathematically described as a one-dimensional finite non-homogenous chain, as illustrated in Figure 1. The model includes a heavy mass, denoted by M, which represents the valve mass together with the mass of the upper element of the spring, and N light masses, each with mass m, i.e. the mass of one spring element. The masses are interconnected in series by linear springs and dampers, denoted by k and λ, respectively, resulting to total spring stiffness of k/N and total internal damping of the spring material of λ/N. In realistic settings of the valve train, the valve spring is preloaded by a static load of kd_0/N, resulting to total preload displacement of d_0 (as shown in Figure 1, middle). Then, the preloaded valve spring is closed by a rigid wall in such a way that the heavy mass interacts with the wall, as illustrated in Figure 1, right. We assume that this interaction is given by Hertzian contact law. This constraint on the displacement of the heavy mass represents the interaction between the valve and valve seat. Regarding the boundary conditions, the Nth light mass is fixed while the heavy mass is subjected to an external displacement $h(t)$, which represents the cam lobe profile, through a flexible connection called tappet, modeled by the spring k_c and damper λ_c.

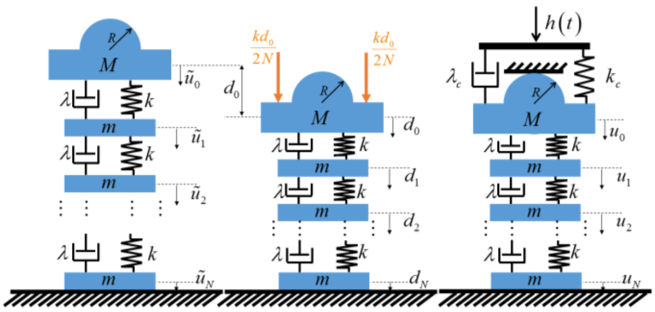

Figure 1: A mathematical model for the valve spring dynamics: free spring (left), preloaded spring in the static state (middle) and the forced spring with the contact constraint (right)

Let u_n denotes the dynamic displacement (relative to the preloaded state) of the nth mass. Thus, the equations of motion for the forced valve spring (Figure 1, right) are given as follows:

$$\begin{aligned} n=0: &\quad M\ddot{u}_0+\lambda(\dot{u}_0-\dot{u}_1)+k(u_0-u_1)+\lambda_c\dot{u}_0+k_cu_0=\alpha[\Delta-u_0]_+^{3/2}-\frac{kd_0}{N}+ \\ &\quad +k_ca_0+[k_ca+\lambda_c\Omega b]\cos(\Omega t)+[k_cb-\lambda_c\Omega a]\sin(\Omega t) \\ 1\le n<N-1: &\quad m\ddot{u}_n+\lambda(2\dot{u}_n-\dot{u}_{n-1}-\dot{u}_{n+1})+k(2u_n-u_{n-1}-u_{n+1})=0 \\ n=N: &\quad u_N=0 \end{aligned} \tag{1}$$

Here in (1), the parameter Δ represents the equilibrium displacement induced by the preload force kd_0/N, resulting to $\Delta = \left(kd_0/\alpha N\right)^{2/3}$, α is a material parameter (depending on the elastic properties and the geometric characteristics of the heavy mass and the wall). The $(+)$ subscript is used to emphasize that the bracketed term is non-zero only if the term inside the bracket is positive and zero otherwise. The parameter Ω describes the frequency of the applied excitation, which was assumed to be given by the first harmonic approximation $h(t) = a_0 + a\cos(\Omega t) + b\sin(\Omega t)$. We assume that the response of each oscillator has a dominant harmonic component with the frequency equal to the frequency of the applied excitation, then, the following single-frequency solution is introduced:

$$u_n = U_0\left(1-\frac{n}{N}\right) + C_n\cos(\Omega t) + S_n\sin(\Omega t) \tag{2}$$

Introducing the suggested solution (2) into (1) and expanding the non-smooth term into Fourier series to obtain static and dynamic components as follows:

$$\left[\Delta - U_0 - C_0\cos(\Omega t) - S_0\sin(\Omega t)\right]_+^{3/2} = \beta_0 + \sum_{r=1}^{\infty}\beta_r\cos(r\Omega t) + \sum_{r=1}^{\infty}\gamma_r\sin(r\Omega t) \tag{3}$$

For the first harmonics, one obtains:

$$\begin{aligned}
\beta_0 &= \frac{\sqrt{2}A^{3/2}}{3\pi}\left[\left(3\sigma^2 - 4\sigma + 1\right)\mathbf{K}(\bar{\sigma}) + 8\sigma\mathbf{E}(\bar{\sigma})\right] \\
\beta_1 &= \frac{2\sqrt{2}A^{3/2}\cos(\phi)}{5\pi}\left[\left(-\sigma^2 + 4\sigma - 3\right)\mathbf{K}(\bar{\sigma}) + 2\left(\sigma^2 + 3\right)\mathbf{E}(\bar{\sigma})\right] \\
\gamma_1 &= -\frac{2\sqrt{2}A^{3/2}\sin(\phi)}{5\pi}\left[\left(-\sigma^2 + 4\sigma - 3\right)\mathbf{K}(\bar{\sigma}) + 2\left(\sigma^2 + 3\right)\mathbf{E}(\bar{\sigma})\right]
\end{aligned} \tag{4}$$

Here we denote $\bar{\sigma} = \sqrt{2\sigma + 2}/2$, $\sigma = \left(\Delta - U_0\right)/A$, $\phi = \arctan\left(-S_0/C_0\right)$, $A = -\sqrt{C_0^{\,2} + S_0^{\,2}}$, $\mathbf{K}(\bar{\sigma})$ represents the complete elliptic integral of the first kind, and $\mathrm{E}(\bar{\sigma})$ is the complete elliptical integral of the second kind.

Substituting equations (2), (3) and (4) into (1) and applying the harmonic balance method, one obtains a set of $(2N+3)$ nonlinear algebraic equations in the unknowns U_0, C_n and S_n, $n = 0,1,\ldots,N$.

Conclusions

In this work, we introduce a strongly nonlinear mathematical model to describe the dynamics of the preloaded valve spring taking into account the valve-valve seat interaction, which has been described by the essentially nonlinear (nonlinearizable) Hertzian contact law. The dynamics of the chain is analytically studied under the assumption that the response of each oscillator has a dominant harmonic component with a frequency equal to the excitation frequency. Depending on the location of the applied frequency in the dispersion curve of the linear chain, the model under consideration reveals two different periodic responses, namely, propagating and edge breathers. The propagating breathers have main frequency in the propagation zone, and are characterized by weak localization, in which the energy can spread among all the masses. The edge breathers (we referred to them as edge states) possess frequency in the attenuation zone, these states are characterized by strong localization at the forced end of the chain, i.e. maximal concentration of energy in the excited mass against small-amplitude oscillations in the rest of the chain. This implies that in the edge states regime, the coupling between the zeroth and first masses can be removed, and the strongly nonlinear forced oscillator can be considered as a single DOF that excites the linear chain. Comparison of the analytical solution with numerical simulations and experimental test results yields close agreement.

References

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