Speed effects on vibration and collapse of slender structures under moving loads

G.H.M. van der Heijden*, X.W. Zhao*

*Department of Civil, Environmental and Geomatic Engineering
University College London, Gower Street, London WC1E 6BT, UK

Summary. We study the nonlinear dynamics of one-dimensional slender structures such as beams and arches carrying a moving load. At resonant speeds the vibrations of curved beams are significantly suppressed, but large deformations are found to have a detuning effect. Arches are found to have different buckling (failure) modes depending on the depth of the arch and the speed of the traversing load.

Modelling

We study the nonlinear dynamics of one-dimensional slender structures such as beams and arches carrying a moving load. Geometrically-exact rod theory is used to model the structure, which is allowed to undergo arbitrary three-dimensional flexural and twisting deformations.

Results and conclusions

The exact natural frequencies ω_n (for symmetric or anti-symmetric modes) of a hinged curved horizontal beam with subtended angle α are given by

$$\omega_n = \mu_n \sqrt{\frac{EI}{\rho A L^4}}, \qquad \text{where} \qquad \mu_n = \frac{n\pi \left|(n\pi)^2 - \alpha^2\right|}{\sqrt{(n\pi)^2 + (1+\nu)\alpha^2}} \qquad (n = 1, 2, ...)$$

(ignoring rotational inertia). Here EI is the bending stiffness, A the cross-sectional area, ρ the density, ν Poisson's ratio and L the length of the beam. The critical resonance speed is defined as the speed at which the maximum midspan deflection occurs when the load leaves the beam. It is given by

$$v_{\text{crit}} = \frac{\omega_1 L}{\pi}$$

(i.e., the fundamental period is twice the passage time).

We find that nonlinearity due to large deformations has a detuning effect on resonances, as can be seen in midspan deflection plots for different values of α (see Figure 1).

At resonant speeds v we find cancellation of vibrations even at large deformations (see Figure 2).

Arches are found to have different buckling (failure) modes depending on the depth of the arch and the speed of the traversing load.

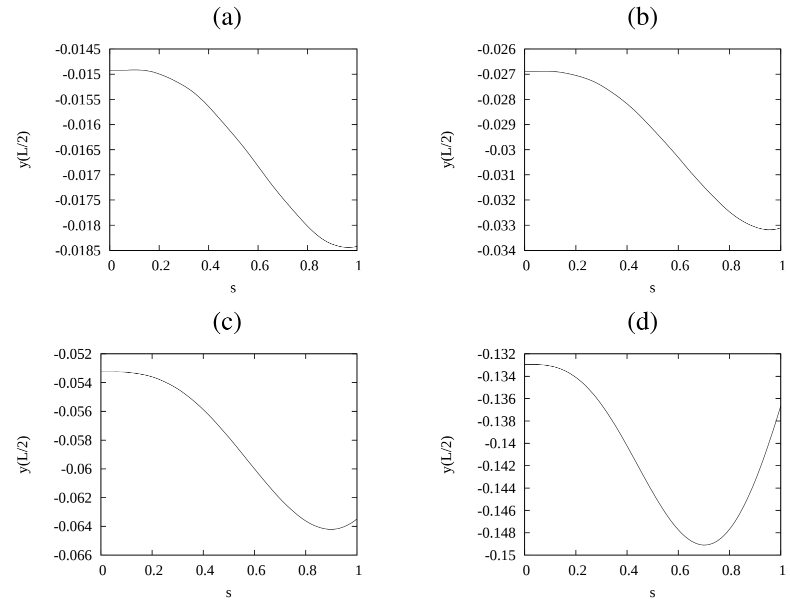

Figure 1: Detuning effect due to large deformations: (a) $\alpha = 50°$, (b) $\alpha = 90°$, (c) $\alpha = 120°$, (d) $\alpha = 150°$. The magnitude of the moving load is $F = 0.1$ (in units of EI/L^2). s is the instantaneous position of the load, normalised by L.

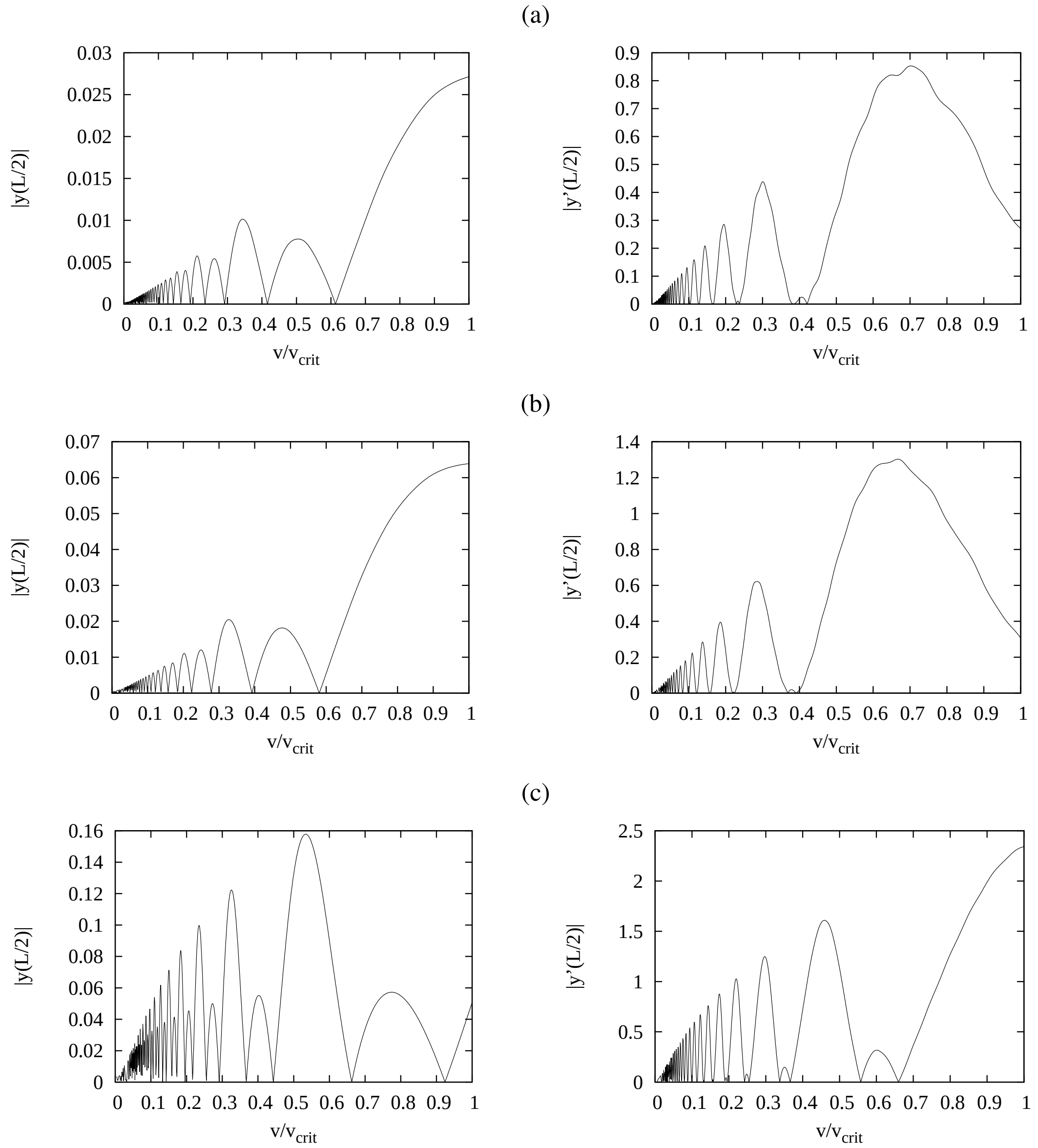

Figure 2: Cancellation of vertical midspan deflection (left) and velocity (right) of a horizontal curved beam with moving load $F = 1.0$ (in units of EI/L^2) at critical speeds v/v_{crit}. (a) $\alpha = 30^\circ$, (b) $\alpha = 90^\circ$, (c) $\alpha = 150^\circ$.