An Input-Output Approach Towards Synchronization Under Communication Constraints

Jijju Thomas[*,†,‡], Erik Steur[*], Laurentiu Hetel[†], Christophe Fiter[†], Jean-Pierre Richard[†,‡], Nathan van de Wouw[*,⋆]
[*] *Department of Mechanical Engineering, Eindhoven University of Technology, The Netherlands*
[†] *Ecole Centrale de Lille/Université de Lille, CRIStAL UMR CNRS 9189, 59650 Villeneuve d'Ascq, France*
[‡] *Inria Lille, Villeneuve d'Ascq, France*
[⋆] *Department of Civil, Environmental and Geo-Engineering, University of Minnesota, Minneapolis, MN 55455 USA.*

Summary. This work presents a novel approach towards synchronization analysis of nonlinear systems, diffusively coupled via a networked communication channel. The system under consideration is a two-agent nonlinear system, under the constraint that information is transmitted between the two systems using an aperiodic communication strategy. The system setting is remodelled as the feedback-interconnection of a continuous-time system, and an operator that captures the perturbations introduced by communication constraints. By studying the properties of the remodelled system, i.e., the feedback-interconnection, in the framework of Dissipativity Theory, we provide a novel stability criterion that guarantees exponential synchronization.

Introduction

In many natural and practical circumstances, the phenomenon of synchronization has caught the attention of researchers and scientists from various fields. Typical examples include flashing fireflies, firing neurons, etc. In control theory, synchronization is a topic of interest in areas such as master-slave synchronization of nonlinear systems [Nijmeijer and Mareels, 1997]. In practical applications such as cooperative control of multi-agent systems [Olfati-Saber et al., 2007], synchronization can be analyzed with a control theoretic approach for networks of nonlinear systems [Pogromsky et al., 2002]. In such contexts, synchronization problems become more complex due to uncertainties introduced via the networked communication channel. For example, delays introduced in the network increase the complexity of synchronization problems [Steur and Nijmeijer, 2011]. In addition to delays, individual systems could be connected via sampled-data coupling.

In recent years, master-slave synchronization problems of linear sampled-data systems have been studied, and different approaches have been proposed to study the relation between sampling period, coupling strength, and synchronization [Hua et al., 2015]. In existing results, it is typically considered that individual systems have the same sampling frequency. However, in practical scenarios, individual systems usually transmit information at different frequencies over a network, depending upon the communication channel, data traffic, etc. In this work, we consider a bidirectionally coupled, sampled-data, two-agent nonlinear system, wherein individual systems transmit information over a networked communication channel, at possibly different, aperiodic frequencies.

Problem Statement

We consider a two-agent interconnected system as shown in Figure 1a, wherein the dynamics of individual sub-system Σ_i, $i = 1, 2$, is of relative degree one, and is given by

$$\begin{aligned} \dot{x}_i(t) &= f(x_i(t)) + Bu_i(t), \\ y_i(t) &= Cx_i(t), \quad i = 1, 2, \end{aligned} \tag{1}$$

where $x_i \in \mathbb{R}^n$, $u_i, y_i \in \mathbb{R}^m$ are the state, input, and output, respectively. The function $f\colon \mathbb{R}^n \mapsto \mathbb{R}^n$ is a sufficiently smooth vector field, and B and C are matrices with appropriate dimensions, with $CB =: b \in \mathbb{R}^{m\times m}$, b being positive definite and without loss of generality, diagonal. The i^{th} output is transmitted to the j^{th} sub-system only at instants given by the sequence $s^i_{k+1} = s^i_k + h^i_k, h^i_k \in [\underline{h}_i, \bar{h}_i], k \in \mathbb{N}, i = 1, 2$. Without loss of generality, we consider $s^i_0 = 0$, $i = 1, 2$.

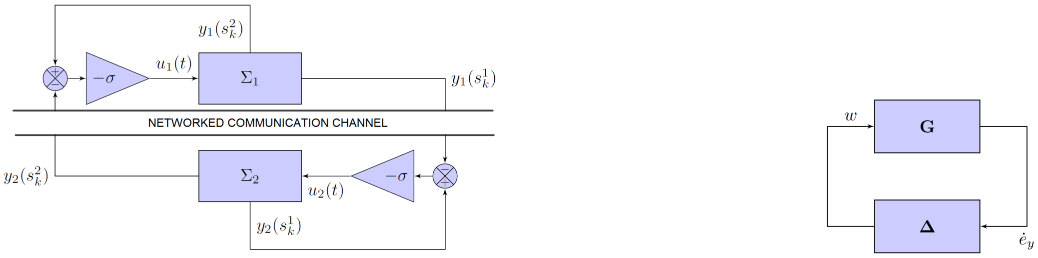
Assumption 1: The i^{th} sub-system has access to local output information at time instants $t = s^j_k, j \neq i, k \in \mathbb{N}$.

The aforementioned assumption reflects a practical scenario wherein individual systems can be sampled locally at a high frequency, but data transmission over a networked communication channel occurs at a lower frequency, depending on network induced constraints or requirements. Exploiting this assumption, we have that $\Sigma_i, i = 1, 2$, has access to local information at instants $s^j_k, j \neq i$. Consequently, the synchronizing coupling is designed as

$$\begin{aligned} u_1(t) &= -\sigma(y_1(s^2_k) - y_2(s^2_k)), \forall t \in [s^2_k, s^2_{k+1}), \\ u_2(t) &= -\sigma(y_2(s^1_k) - y_1(s^1_k)), \forall t \in [s^1_k, s^1_{k+1}), \end{aligned} \tag{2}$$

where $\sigma \in \mathbb{R}_+$ is the constant coupling strength. Since $CB > 0$, there exists a coordinate transformation so that the i^{th} sub-system dynamics are given by

$$\Sigma_i : \begin{cases} \dot{z}_i(t) &= q(z_i(t), y_i(t)), \\ \dot{y}_i(t) &= a(z_i(t), y_i(t)) + bu_i(t), i = 1, 2, \end{cases} \tag{3}$$



(a) Bidirectionally coupled systems Σ_1 and Σ_2 under asynchronous sampled-data transmission.

(b) System shown in Figure 1a represented as a feedback-interconnection.

Figure 1: (a) Systems Σ_1 and Σ_2 coupled via a networked communication channel and, (b) an equivalent feedback-interconnection representation.

where $z_i \in \mathbb{R}^{n-m}$, $u_i, y_i \in \mathbb{R}^m$, $q\colon \mathbb{R}^{n-m} \times \mathbb{R}^m \mapsto \mathbb{R}^{n-m}$, and $a\colon \mathbb{R}^{n-m} \times \mathbb{R}^m \mapsto \mathbb{R}^m$.

Assumption 2: The solution of the closed-loop system (3), (2) is ultimately bounded.

Definition 1: The bidirectionally coupled system given by (3), (2) is said to synchronize if $\lim_{t\to\infty} \|e(t)\| \to 0$, where $e(t) = \begin{bmatrix} e_y^T(t) & e_z^T(t) \end{bmatrix}^T$, and $e_y(t) = y_1(t) - y_2(t)$, $e_z(t) = z_1(t) - z_2(t)$, for any initial conditions $(z_1(0), y_1(0))$ and $(z_2(0), y_2(0))$.

Assumption 3: (Demidovich Condition [Pavlov et al., 2005]) There exists a positive definite matrix $P \in \mathbb{R}^{(n-m)\times(n-m)}$, such that the internal state dynamics given by $\dot{z}_i(t) = q(z_i(t), y_i(t))$, $i = 1, 2$, satisfies

$$P\frac{\partial q}{\partial z_i}(z_i, y_i) + \frac{\partial q^T}{\partial z_i}(z_i, y_i)P \leq -\delta I_{n-m}, P = P^T > 0, \delta > 0. \tag{4}$$

In this work, we provide conditions that guarantee exponential synchronization of the coupled system (3), (2).

Main Result

The system (3), (2) shown in Figure 1a is remodelled such that the effects introduced due to aperiodic sampling are decoupled from the continuous-time network, as shown in Figure 1b. In Figure 1b, the operator $\mathbf{G}$ represents the dynamics of the system (1) in the absence of sampling, i.e., $\mathbf{G}$ represents a 'continuous-time' version of the system (3), (2). Additionally, the operator $\boldsymbol{\Delta}$ captures the error induced in the system due to asynchronous sampling. Consequently, the feedback-interconnection shown in Figure 1b represents the bidirectionally coupled, sampled-data system configuration shown in Figure 1a, by considering the effects of sampling as a perturbation to the continuous-time system operator $\mathbf{G}$. By studying the properties of the feedback-interconnection $\mathbf{G} - \boldsymbol{\Delta}$, we provide conditions that guarantee exponential stability of the error dynamics $e(t)$, i.e., exponential synchronization of the system (3), (2). We adapt the result provided in [Omran et al., 2016], wherein a dissipativity based framework was employed to prove asymptotic stability of nonlinear systems with aperiodic sampled-data control. The properties of operator $\boldsymbol{\Delta}$ are characterized by a function $\mathcal{S}$ that satisfies

$$\int_0^t \mathcal{S}(\theta, e_y(\theta), \phi\{y_1(\theta), y_2(\theta), z_1(\theta), z_2(\theta)\}, w(\theta))d\theta \leq 0, \forall t \geq 0, \tag{5}$$

where

$$\begin{aligned} &\mathcal{S}(t, e_y(t), \phi\{y_1(t), y_2(t), z_1(t), z_2(t)\}, w(t)) \\ &\quad := w^T(t)Rw(t) - \gamma^2(\phi(y_1(t), y_2(t), z_1(t), z_2(t)) - 2b\sigma e_y(t) + b\sigma w(t))^T R(\phi(y_1(t), y_2(t), z_1(t), z_2(t)) \\ &\quad\quad -2b\sigma e_y(t) + b\sigma w(t)), \end{aligned} \tag{6}$$

with $\gamma^2 = \frac{4(\bar{h}_1^2 + \bar{h}_2^2)}{\pi^2}$, $w(t) = (\boldsymbol{\Delta}\dot{e}_y)(t)$, and $\phi(y_1(t), y_2(t), z_1(t), z_2(t)) = a(z_1(t), y_1(t)) - a(z_2(t), y_2(t))$. For a positive definite storage function V, if the condition

$$\dot{V}(e(t)) + \alpha V(e(t)) \leq e^{-\alpha t}\mathcal{S}(t, e_y(t), \phi\{y_1(t), y_2(t), z_1(t), z_2(t)\}, w(t)), \forall t \geq 0, \alpha > 0, \tag{7}$$

holds, then the system setting given by (3), (2), synchronizes exponentially with a decay rate of at least $\alpha/2$.

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