Probabilistic response of a vibration energy harvester for realistic torsional vibrations

Panagiotis Alevras*, Ben Gunn** and Stephanos Theodossiades**
**Department of Mechanical Engineering, School of Engineering, University of Birmingham, Birmingham, UK*
***Wolfson School of Mechanical, Electrical and Manufacturing Engineering, Loughborough University, Loughborough, UK*

Summary. A nonlinear vibration energy harvester for converting energy from the torsional oscillations of an automotive powertrain is considered under random engine running speeds. Engine data recorded form several real-life routes are used to describe the spectral properties of the powertrain vibrations, and particularly their frequency distribution in city and highway driving cycles. Joint response Probability Density Functions of the stochastic coupled electromechanical oscillator are calculated with a numerical Path Integration approach. The impact of noise on the mean power output is assessed by comparing the resulting random power output statistics with deterministic expressions based on the mean engine speed.

Introduction

Vibration energy harvesting is a recent concept that involves scavenging energy from ambient mechanical vibrations and converting it to useful electricity to power small electronic devices, such as sensors and wireless data transmitters [1]. These systems are designed such that mechanical power take-off is optimised, and this is usually achieved by establishing high-amplitude response to the exciting vibrations of the host system, such as resonant linear response or broadband high energy response in the case of nonlinear harvesters [2]. However, vibrations in structures and machines are usually the result of complex interactions between nonlinearly coupled components, exhibiting a diverse interaction with system modes and the inherent randomness of environmental forces. This is particularly pertinent to automotive vibrations whereby linear and torsional vibrations of the powertrain components are subject to a multi-factor dependence on environmental conditions, road surface conditions, driving behaviour, route and mode of driving (city/highway). As a result, harvesters are essentially required to capture energy from vibrations that cannot be adequately described by conventional deterministic approaches.
Recently, a concept for harvesting energy from the torsional vibrations of a propulsion shaft has been introduced [3]. In this paper, a stochastic framework for calculating this device's mean harvested power is presented. Randomness of the powertrain vibrations frequency is modelled by exploiting the direct link of the dominant vibration frequency to the engine speed and the engine architecture. Transient and steady-state driving scenarios are considered through a statistical analysis of engine speeds recorded in real-life driving scenarios. Expressions of the spectral distributions of the input vibrations are then used within an advanced computational framework to accurately calculate joint Probability Density Functions (PDF) of the harvester's mechanical and electrical generalised variables.

Problem Formulation

Consider a rotational vibration energy harvester as the one shown in Figure 1(a). This concept has been recently introduced for harvesting energy from the torsional vibrations of propulsion shafts [3]. The system has been designed to harvest energy from torsional vibrations rather than the main shaft speed, thereby minimising the impact of the device on the main transmission of power with potentially favourable reduction of the shaft speed fluctuations. The reader is referred to [3] for a detailed description of the device and the ensuing analysis that leads to the following equations of motion:

$$\begin{gathered} J\ddot{\phi} + c_m\dot{\phi} + k_1\phi + k_3\phi^3 - \widehat{\Theta}I = J\ddot{a} \\ L\dot{I} + (R_{load} + R_{int})I + \widehat{\Theta}\dot{\phi} = 0 \end{gathered} \quad (1)$$

where $\ddot{a}$ denotes the shaft vibrations which act as base excitation to the rotational harvester with inertia J, mechanical damping c_m and potential energy $U(\phi) = k_1\phi^2/2 + k_3\phi^4/4$. The electrical properties are denoted by L – inductance, R_{load} – load resistance, R_{int} – coil resistance and $\widehat{\Theta}$ – electromagnetic coupling. Dominant vibration frequencies in rotary applications are linked with orders of the main rotation speed. In 2-cylinder, 2-stroke IC engines for example, powertrain vibrations are dominated by the 2^{nd} engine order. However, as it is shown in Figure 1(b), the engine speed that determines the dominant vibration frequency is a randomly varying quantity (see Figure 1(d)). This data has been recorded in a real driving cycle down the route shown in Figure 1(c). Therefore, the exciting base vibrations can take the following form to consider random modulations of the excitation's phase:

$$\begin{gathered} \ddot{a} = A\cos(q) \\ \dot{q} = \nu + \sigma\zeta(t) \end{gathered} \quad (2)$$

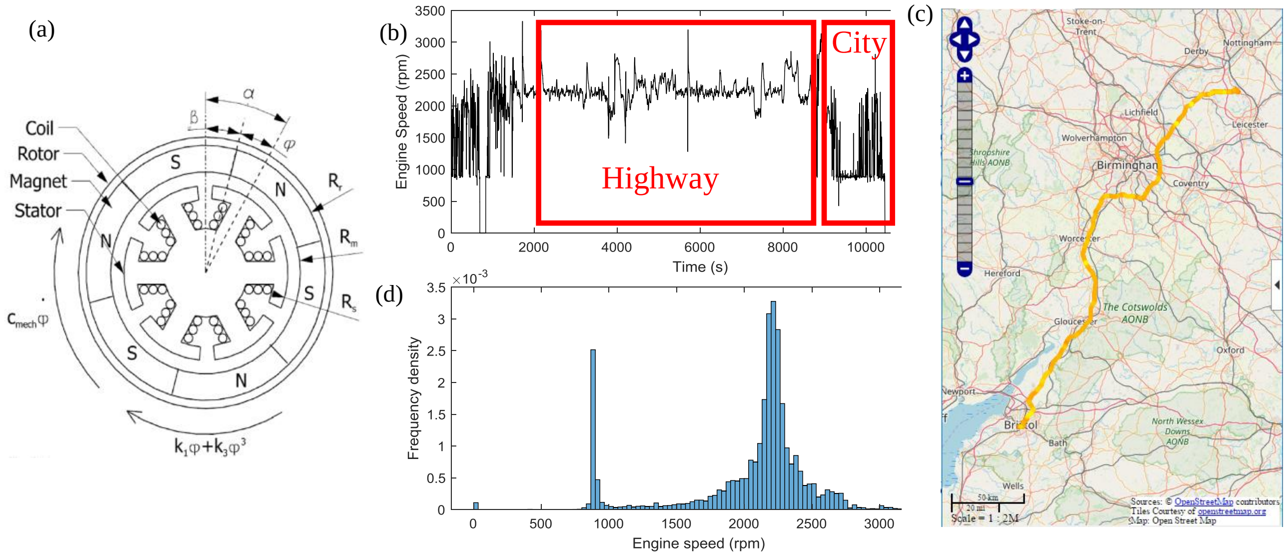

Figure 1: (a) sketch of the rotational vibration energy harvester; (b) engine speed during mixed drive cycle; (c) route map of displayed data; (d) distribution of engine speed in mixed cycle.

where $\zeta(t)$ is a Gaussian delta-correlated white noise process with $\langle\zeta(t)\zeta(t+\tau)\rangle = 2\pi D\delta(\tau)$. Note that randomness is only considered in the phase of the vibrations since the vibration amplitude can be considered to be slow-varying. More explicit representation of the speed's spectral properties can be used by colouring the white noise process with higher order linear filters.

Path Integration

The joint response PDF of the stochastic system described by Eqs (1) and (2) is computed by means of a numerical Path Integration (PI) technique. The response PDF contains probabilistic information for the harvester's kinematic and electrical variables, allowing accurate calculation of the electrical power output probability distribution. This method is based on an iterative approach for calculating in short time steps the response PDF, which at time t can be expressed from the PDF at $t' = t - \Delta t$ exploiting the Markov property of the solution vector and the total probability law [4]:

$$p(\mathbf{z}, q, t) = \int_{state\ space} p(\mathbf{z}, q, t|\mathbf{z}', q', t')p(\mathbf{z}', q', t')dzdq \qquad (3)$$

where $p(\mathbf{z}, q, t|\mathbf{z}', q', t')$ is the transition probability density function (TPD) and $\mathbf{z}$ is the system's state vector. The short time propagation TPD is Gaussian distribution and thus, the TPD may be written as [4]:

$$p(\mathbf{z}, q, t|\mathbf{z}', q', t') = \delta[\mathbf{z} - \mathbf{z}' - \mathbf{r}(\mathbf{z}', q', \Delta t)]\frac{1}{\sqrt{2\pi D\Delta t}}\exp\left\{-\frac{[q - q' - r_4(\mathbf{z}', q', \Delta t\,)]^2}{2D\Delta t}\right\} \qquad (4)$$

where δ is the Dirac delta function and $\mathbf{r}\,(z', q', \Delta t)$, $i = 1,2,3,\ldots$ are 4th order Runge-Kutta increments of $\mathbf{z}$ and q. Iterative application of Eqs (3) and (4) lead to the computation of the joint response PDF, whereby standard rules are applied to compute the marginal PDF of the current and the electrical power.

Discussion

Preliminary calculations have shown that randomness has a significant impact on the mean power output compared with using the average engine speed in deterministic models. This is particularly important for vibrations in the frequency range in-between the deterministic saddle-node bifurcations, where multiple attractors co-exist. Early results indicate that randomness particularly affects the establishment of high energy solution branches as the solution approaches the jump-down bifurcations. This is due to the shrinking area of the deterministic basin of attraction which makes the high-amplitude desired solution more sensitive to random fluctuations. Further analysis using the described procedure will reveal the full impact of randomness on the harvested electrical power.

References

[1] Stephen N. G. (2206) On energy harvesting from ambient vibration. *J. Sound Vib.* **293** 409–425.

[2] Alevras P., Theodossiades S., Rahnejat H. (2018) On the dynamics of a nonlinear energy harvester with multiple resonant zones. Nonlin Dyn **92**(3) 1271-1286.

[3] Gunn B. E., Theodossiades S., Rothberg S. J. (2019) A Nonlinear Concept of Electromagnetic Energy Harvester for Rotational Applications. *J. Vib. Acoust.* **141**(3): 031005 (13 pages).
[4] Yurchenko D., Naess A., Alevras P. (2013) Pendulum's rotational motion governed by a stochastic Mathieu equation. *Probabilist. Eng. Mech.* **31** 12-18.