Nonlinear localisation in a cyclic system with unilateral contact

Björn NIEDERGESÄß*, Aurélien GROLET† and Norbert HOFFMANN *
* *Hamburg University of Technology, Hamburg, Germany*
† *LISPEN, ENSAM Lille, Lille, France*

Summary. The following abstract presents a cyclic structure subjected to unilateral contact in order to investigate nonlinear localisation of vibration. After a brief description of the system, a reduced-order model is derived for numerical analysis. Subsequently, the experimental setup is introduced and the system response to harmonic excitation is depicted. In the nonlinear regime vibration localisation is observed with a high amplitude ratio regarding the beam elements of the structure.

Introduction

Localisation of vibration corresponds to states where the vibration energy is spatially localised to a section of the system so that only this section vibrates in high amplitude. It has been observed in a vast variety of systems either numerically, e.g. for coupled Duffing oscillators [1] and coupled beams [2], or experimentally, e.g. for a microbeam array [3] and a macro mass-spring system [4]. The localisation arises from nonlinear modal interaction, such as 1:1 internal resonances [5]. In this work experimental measurements of localised vibration states are reported in a cyclic structure subjected to unilateral contact.

Presentation of the system and reduced-order model

Presentation of the system

The structure investigated in this paper consists of three clamped-free beams with attached tip masses ordered in a cyclic fashion and coupled through slender connections as depicted in Fig.1. When vibrating in the transverse direction the beams are subjected to unilateral contact enforced at a level between the slender connection and the tip mass. Since the mass of the beams is small compared to the attached tip masses it can be neglected and the system can be regarded as three coupled nonlinear oscillators.

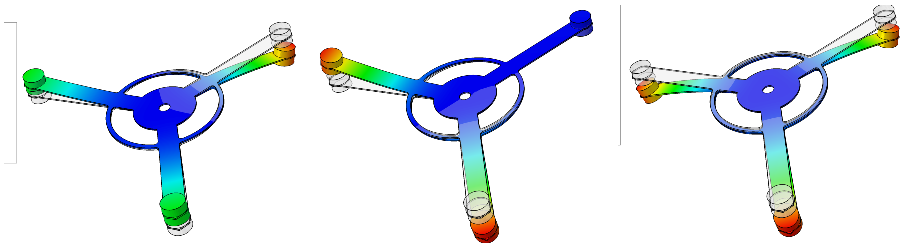

Figure 1: First three linear mode shapes of the considered structure. Note that, due to the symmetry, the first two modes have the same eigen-frequency

Reduced-order model

Based on the FEM mode shapes, a lumped mass model with three degrees of freedom can be derived for numerical analysis. The contact nonlinearity for the i-th oscillator is modeled by a bi-linear force $g(u_i)$: in small amplitude, the beam is not in contact so that the force is zero. When the tip displacement reaches a threshold u_g the effective stiffness increases and tends toward one of a beam with "pinned" boundary condition at the position of the contact. The force can be expressed as a bi-linear function of the tip displacement:

$$g(u_i) = \begin{cases} 0 & \text{if } u_i \leq u_g \\ K(u_i - u_g) & \text{if } u_i \geq u_g, \end{cases} \tag{1}$$

where K is the contact stiffness and u_g is the tip displacement at the moment of contact. In order to find periodic solutions to the equations of motion, the harmonic balance method (HBM) is applied. The solution is searched for a truncated Fourier series and the coefficients are obtained by solving a system of nonlinear algebraic equation. This system will be solved by the asymptotic numeric method (ANM). Both methods (HBM and ANM) are implemented in a common framework in the MANLAB package, along with the stability computation of the periodic solutions [6]. The equations of motion are written in a quadratic form for MANLAB and the required regularisation function g_{reg} of the unilateral contact force is obtained over a hyperbola branch which is defined by the following quadratic equation:

$$g_{reg}[g_{reg} - K(u_i - u_g)] = \epsilon, \tag{2}$$

where a smaller regularisation parameter ϵ indicates a hyperbola closer to the piecewise curve.

Experimental setup and results

Description of the experiment

In the experimental setup the structure depicted in Fig 1 and a contact disk share a common base, which is mounted on two slender plates. The base is excited by a shaker (TIRA TV 51140-M) and vibrates harmonically due to the mode of the supporting plates. Therefor a harmonic excitation signal is fed in the shaker in open loop, processed by an ADwin-Gold control box with a sampling frequency of 4kHz. To measure the response of the system accelerometers are glued to the base, as well as to the three tip masses. When the system is excited close to the first eigenfrequency of the structure the coupled beams start touching the contact disk.

Results

Fig. 2 depicts the measured response of the base and the three tip masses m_i of the structure while excited with a frequency of 11.35 Hz. At this excitation level the structure gets in unilateral contact with the disk which amounts in a disturbance of the harmonic excitation depicted in Panel (a). The nonlinear localisation of the structure is depicted in Panel (b) where the amplitude ratio A_1/A_3 is ~ 9.56 ($A_1/A_2 \sim 7.32$ respectively).

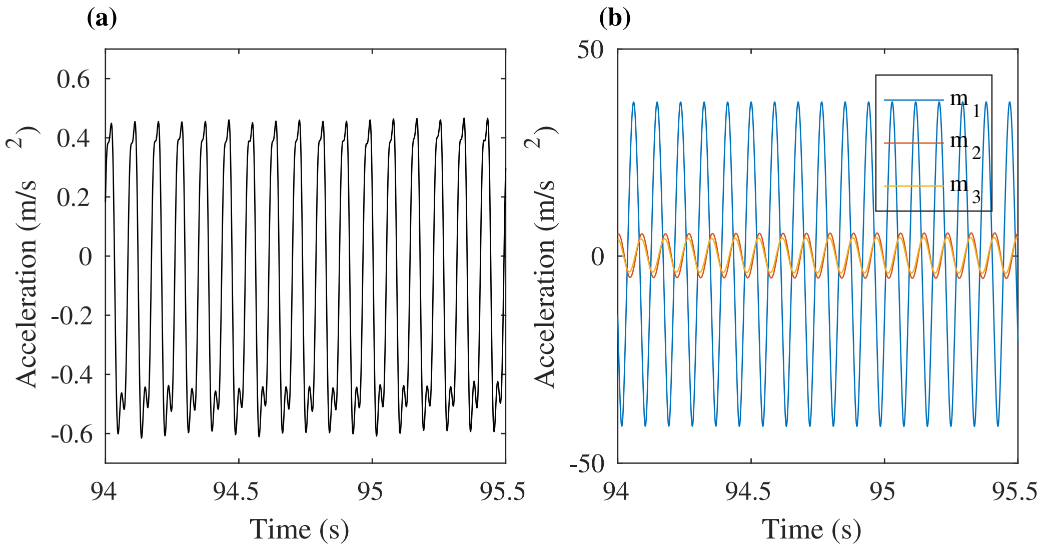

Figure 2: Nonlinear localisation at 11.35Hz : a) Base excitation; b) Response at the tip mass m_i of the structure. m_2 and m_3 vibrate in low amplitude while m_1 vibrates in high amplitude

Conclusions

The abstract considers nonlinear vibrations of a cyclic system subjected to contact nonlinearity. The investigated system can be regarded as a simple qualitative model of more complex structures such as bladed disk in airplane engines. Numerical results based on a reduced-order model indicate that homogeneous solutions can lose their stability to lead to localised states of vibration. The nonlinear localisation is observed in the experimental investigation of the structure.

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