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Basins of attraction for the model of rotating hub with two pendulums

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Summary. The goal of this paper is to study nonlinear dynamics of the system consisting of two pendulums attached to a hub rotating in the horizontal plane. Equations of motion are derived from Lagrange equations of the second kind. Next, the equations are transformed to the system of six first-order ordinary differential equations. Based on the set of first-order differential equations of motion basins of attraction are calculated. Moreover, the synchronisation phenomenon is studied in case of symmetric and nonsymmetric pendulums.

Introduction

Rotating structures have many applications in mechanical and aerospace engineering, like for example arms of industrial robots, wind turbines, helicopter rotors and jet engines. Additionally, pendulums are very well known as a dampers for instance in high buildings and helicopter rotor dampers. There are many publications, which are concentrated on pendulums, like a paper about a model consisting of chains of nonlinear coupled pendulums subjected to harmonic excitations [1]. Authors studied nonlinear dynamics and synchronisation phenomenon for that theoretical model, they examined an influence of initial conditions as well as an influence of horizontal excitation on motion of the chain system. Most of the paper about dynamics of pendulums are focused on pendulums rotating only in a vertical plane, while a different approach is presented in [2]. Authors presented numerical results for a model of a hub with pendulums rotating in the horizontal plane, they proved that the model is strongly nonlinear and even chaotic motion may appear. Authors linearised the problem and solved such equations analytically for small oscillations. Whereas, analysis of basin of attraction are very useful, specially in case of strongly nonlinear systems. They consist regions of all the initial conditions that converge to associated attractors forward in time. Analyzing the basins of attraction we can inspect the impact of initial conditions on the global behaviour of a dynamical system [3].
The aim of this paper is to study the dynamics and synchronisation phenomenon in a system composed of a rotating hub with two pendulums. Basins of attractions are calculated to study excited vibrations of the system. Additionally, the synchronisation phenomenon in a case of symmetric and nonsymmetric system and bifurcation scenario is study as well.

Model and equations of motion

The analyzed model consists of the two pendulums attached to a rigid hub rotating in the horizontal plane. Mathematical pendulums are connected to the hub by joints treated as a flapping hinge with nonlinear Duffing's type spring and a linear viscous damper.

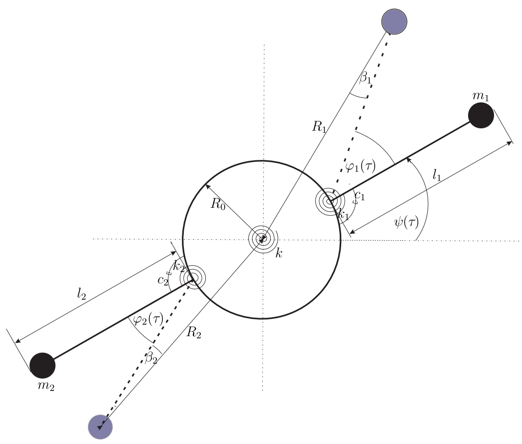

Figure 1: Model of the hub with two pendulums rotating in the horizontal plane; top view

The hub is described by radius R_0 and mass moment of inertia J_0, while m_j and l_j are mass and length of the pendulums, c_j is a viscous damping coefficient and k_j, k_j^*, k and k^* are the nonlinear spring coefficients, where $j = 1, 2$. Two types of nonlinear springs, first type in the hinges and second type spring connecting the hub with background are used in the model. Additionally we assume that, the hub may oscillate or rotate in the horizontal plane, while pendulums may oscillate and also rotate around the hinge in their relative motion. The gravity force does not influence the dynamics of the system, because of the hub rotation in a horizontal plane. Motion of the hub is represented by angle of rotation ψ, while variables φ_j are a coordinates of relative motion of the pendulum with respect to the hub.

The differential equations of motion are derived on the basis of Lagrange equations of the second kind and written in the dimensionless form as shown below:

$$
\begin{gathered}
(1+\gamma_1+\gamma_2)\,\ddot{\psi}+\zeta_h\dot{\psi}+2\frac{\gamma_1}{\delta_1}\dot{\delta}_1\dot{\psi}+2\frac{\gamma_2}{\delta_2}\dot{\delta}_2\dot{\psi}+\frac{\gamma_1}{\delta_1}\cos\beta_1\ddot{\varphi}_1+\frac{\gamma_2}{\delta_2}\cos\beta_2\ddot{\varphi}_2\\
+\frac{\gamma_1}{\delta_1^2}\cos\beta_1\dot{\delta}_1\dot{\varphi}_1+\frac{\gamma_2}{\delta_2^2}\cos\beta_2\dot{\delta}_2\dot{\varphi}_2+\frac{\gamma_1}{\delta_1}\frac{d}{d\tau}(\cos\beta_1)\,\dot{\varphi}_1+\frac{\gamma_2}{\delta_2}\frac{d}{d\tau}(\cos\beta_2)\,\dot{\varphi}_2+\kappa_h\psi+\kappa_h\psi^3=\mu\\
\ddot{\varphi}_1+\delta_1\cos\beta_1\ddot{\psi}+\delta_1\frac{d}{d\tau}(\cos\beta_1)\,\dot{\psi}+\cos\beta_1\dot{\delta}_1\dot{\psi}-\delta_1\frac{d\delta_1}{d\varphi_1}\dot{\psi}^2\\
-\cos\beta_1\dot{\delta}_1\dot{\psi}-\delta_1\frac{d}{d\tau}(\cos\beta_1)\,\dot{\psi}+\zeta_1\dot{\varphi}_1+\omega_{01}^2\varphi_1+\kappa_1\omega_{01}^2\varphi_1^3=0\\
\ddot{\varphi}_2+\delta_2\cos\beta_2\ddot{\psi}+\delta_2\frac{d}{d\tau}(\cos\beta_2)\,\dot{\psi}+\cos\beta_2\dot{\delta}_2\dot{\psi}-\delta_2\frac{d\delta_2}{d\varphi_2}\dot{\psi}^2\\
-\cos\beta_2\dot{\delta}_2\dot{\psi}-\delta_2\frac{d}{d\tau}(\cos\beta_2)\,\dot{\psi}+\zeta_2\dot{\varphi}_2+\omega_{02}^2\varphi_2+\kappa_2\omega_{02}^2\varphi_2^3=0,
\end{gathered}
\tag{1}
$$

where μ is external harmonic torque supplied to the hub, defined as: $\mu = \rho\cos(\omega\tau)$. The equations of motion (1) are strongly nonlinear and coupled by inertia and nonlinear geometric terms. First equation describes the motion of the hub, while two other equations represent the motion of each pendulum.

Results

The set of equations have been transformed to the system of six first-order ordinary differential equations and the following relations have been introduced: $\psi = y_o, \dot{\psi} = y_1, \varphi_1 = y_2, \dot{\varphi}_1 = y_3, \varphi_2 = y_4$ and $\dot{\varphi}_2 = y_5$. In the numerical calculation it is assumed that the pendulum No. 1 is slightly shorter then second pendulum. Firstly, the resonance curves and trajectories of the system attractors are computed. Next, basins of attraction are obtained, which is very time consuming numerical procedure.

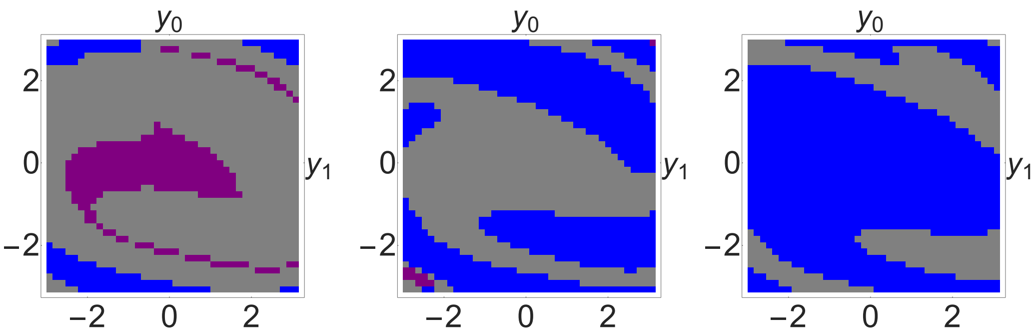

Figure 2: Basin cross-section of the hub, segment with $y_{2-5} = 0.075$ for the excitation frequency a) $\omega = 1.4$, b) $\omega = 1.5$ and c) $\omega = 1.6$

In Fig. 2 we are able to see basin cross-section for the hub, where initial states of pendulums are close to zero. For the frequency $\omega = 1.4$ the gray basin takes most of the area, for higher values of ω the blue basin increases significantly. Furthermore, results on Fig. 2 shows that the motion of the hub in most of the cases converge to gray and blue attractors, while purple attractor has impact motion of pendulums. Blue, purple and gray attractors are corresponding to periodic motion.

Final remarks

The strongly nonlinear model of the hub rotating in the horizontal plane with two pendulums is studied. The resonance curves and trajectories of the system attractors are computed as well as basins of attraction. The obtained results give us more accurate understanding of the dynamics of the analyzed model.

Acknowledgment

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