Parametric resonance in floating bodies - Comparing monochromatic and polychromatic input waves

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Summary. Parametric resonance in floating bodies is well known for large amplitude pitch and roll oscillations with a slower period than the wave induced motions. The pitch and roll dynamics exhibit characteristics of the damped Mathieu equation, due the restoring torque being dependent on the heave postion, which is oscillating in response to the input waves. Therefore, when ocean waves are approximated as a sinusoid, the monochromatic input wave can cause parametric resonance to occur in the system when the input frequency is around twice the pitch/roll natural frequency. However, in reality ocean waves contain a spectrum of frequencies. This paper examines the motion of floating bodies subject to input waves represented by sinusoids and by standard ocean wave spectra. The occurance of parametric resonance is compared for these two cases. A test case, considering a spar-type heaving cylinder from the literature, is presented, showing the resulting parametrically induced pitch and roll motions for the different input waves.

Parametric resonance in floating bodies

Parametric resonance is caused by the time-varying changes in the parameters of a dynamical system, resulting in an exponential increase in oscillation amplitude [1]. Parametric resonance is known to cause large amplitude pitch and roll oscillations in floating bodies. This phenomena was first noted by Froude in 1861 [2], where large amplitude roll motions were observed for a ship when the input wave frequency was half the roll frequency. Parametric pitch/roll is relevant in a number of fields, such as shipping [3], spar-buoy platforms [4], marine based wireless sensor networking [5] and offshore renewable energy [6].

Dynamics

The pitch and roll dynamics of a floating body (we consider a heaving, spar-type cylinder as depicted in Figure 2-(b)) can be well approximated by the Damped Mathieu Equation [1]:

$$\ddot{x}(t) + b\dot{x}(t) + a(t)x(t) = 0. \tag{1}$$

Mechanically, this equation describes the motion, $x(t)$, of a mass-spring-damper system with a time-varying spring stiffness term, $a(t)$. For harmonic variation of the parameter, $a(t)$, the system is known to become unstable at certain frequencies and amplitude thresholds. For a floating body, the spring stiffness in the pitch and roll degrees of freedom, depends on the heave position, which varies in time due to wave induced oscillations. Therefore, the wave induced heave motion of a floating body can trigger parametric resonance in the pitch and roll modes of motion. This is depicted in Figure 1, showing the heave and pitch motions of a floating cylinder subjected to a sinusoidal input wave with a frequency twice the pitch natural frequency.

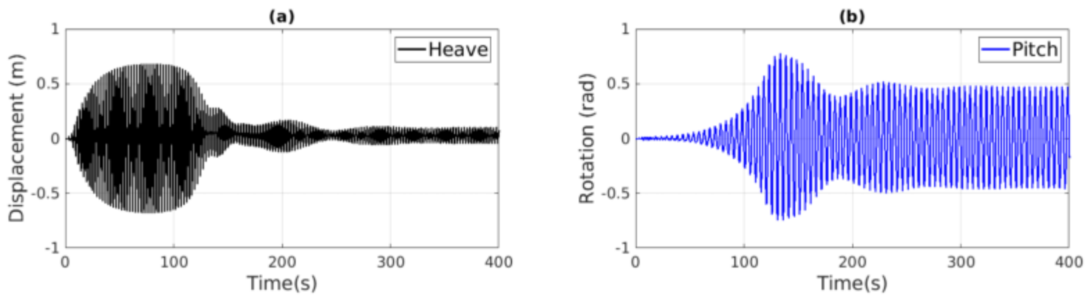

Figure 1: The heave and pitch response of a floating cylinder, subject to an input sinusoidal wave, showing the transfer of energy from heave to pitch

Ocean waves

While the analysis of parametric resonance and Mathieu instability is well known for sinusoidal waves, real ocean waves are stochastic [7]. For engineering purposes, ocean waves are often approximated by Fourier analysis, described by a wave spectrum, with the sea surface well represented by a linear superposition of harmonic components. Figure 2-(a) shows standard wave spectra, JONSWAP and Pierson-Moskowitz, whose shape and bandwidth depends on the wind speed and fetch distance over which the wind blows.

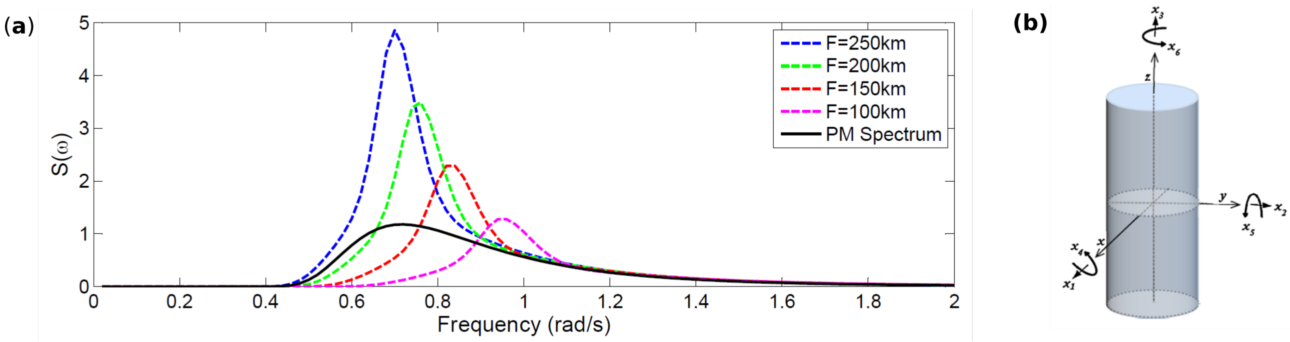

Figure 2: (a) Comparison of the JONSWAP spectrum for varying fetch, F, distance (dashed lines) against the Pierson-Moskowitz (PM) spectrum for a 12m/s wind speed (from [5]) (b) Schematic of the generic spar-type structure in the test case (from [9]).

Test case

As an illustrative example, we consider a heaving, spar-type cylinder, as depicted in Figure 2-(b), whose dynamic instability was previosuly examined in [8, 9, 10]. The pitch/roll natural frequency of the cylinder is twice the pitch/roll natural frequency which makes the cylinder particularly prone to parametric resonance. Numerical simulations of the cylinder motion are presented, considering a range of monochromatic and polychromatic wave inputs and the occurance of parametric resonance in the different wave regimes compared. We compare the occurance of parametric resonance in floating bodies, for experiments considering single frequency or multi-frequency input waves. For the multi-frequency spectra, the effect of the bandwidth is also investigated, comparing narrowband with broadband spectra.

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