Nonlinear Motions of a Self-adaptive Resonator

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Summary. This study reports some experimental results of a beam-slider vibration system which is often called the self-adaptive resonator. It is said to increase the efficient frequency bandwidth of a vibration-based energy harvester. In this study, excitation experiments were carried out focusing on the interaction between the vibrating beam and the slider. The experimental results show that a certain mode of the beam resonates as a result of the slider moving along the beam under excitation at a certain frequency. Some of the results also show the effect of nonlinear modal interactions through the slider movement.

Introduction

A self-adaptive resonator consisting of a beam and a slider was introduced to the field of vibration energy harvesting by Miller[1] as a solution to the problem of narrow resonance frequency range. The proposed system is said to achieve resonance at multiple frequencies, thanks to the slider which, under the condition of excitation, moves along the beam to such a position that the beam will resonate. To discuss the dynamics of the system, we must mention that behaviors of a vibrating beam-slider structure was studied well before it was incorporated to energy harvesting. Recent studies by Lyu[2] report that the adaptive behavior of the system is influenced by the geometrical nonlinearity of the vibrating beam. In this study we conducted some experiments to find out the important factors to explain the motion of the beam and the slider.

Analytical Studies

Governing equations

Figure 1 shows the image of the analytical model. The horizontal position is denoted by x, and the vertical beam displacement at each position is denoted by W. s stands for the position of the slider, which would be a factor that changes the natural frequency and the beam shape. Hence W is a function of time t, x and s, and the equation of the beam and sliders motion can be described as follows[1].

$$\rho A(\ddot{W}+\ddot{W}_O)+EI\frac{d^4W}{dx^4}+\ddot{W}_O+M\left(\ddot{W}+\ddot{W}_O+\ddot{s}\frac{dW}{dx}+2\dot{s}\frac{d\dot{W}}{dx}+\dot{s}^2\frac{d^2W}{dx^2}\right)\delta(x-s)=0 \tag{1}$$

$$M\ddot{s}+M\left[\left(\ddot{W}+\ddot{W}_O+\ddot{s}\frac{dW}{dx}+2\dot{s}\frac{d\dot{W}}{dx}+\dot{s}^2\frac{d^2W}{dx^2}\right)\frac{dW}{dx}\right]_{x=s}=0 \tag{2}$$

M is the slider's mass, W_O is the excitation term, ρ, A, E and I are the density, cross-sectional area, Young's modulus and second moment of inertia of the beam, respectively.

Theory

In early studies, Thomsen[3] assumed that the slider's mass was light compared to the beam, hence neglected the change of the beam shape. This assumption provides a tendency in the slider's motion, that the slider would move toward the antinode of one of the modes which stands out the most. However, experimental studies report that the self-adaptive behaviors ideal to energy harvesting can be observed mostly when the slider's mass is relatively large. This results in not only the change of the beam shape, but also some unnatural motions of the slider, possibly due to linear and nonlinear couplings between the modes or between the beam and the slider.
Before we move on to the experimental study, we must introduce the dependencies of the natural frequency of the first and second mode for a fixed-fixed beam with a fixed additional mass. Figure 2 shows an example of those relationships with a shared horizontal axis indicating the mass position, with the additional mass being slightly heavier than the beam. According to this figure, depending on the position of the slider, the natural frequency ratio of these two modes may become an integer ratio such as 1 to 2, which may cause nonlinear coupling between the modes..

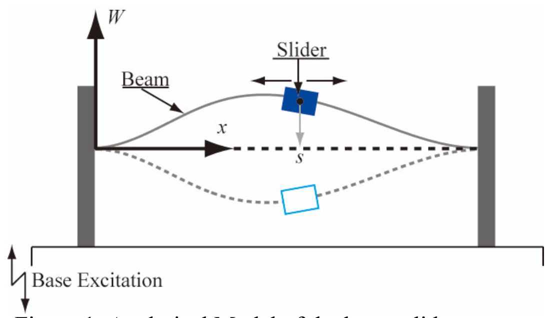

Figure 1: Analytical Model of the beam-slider system.

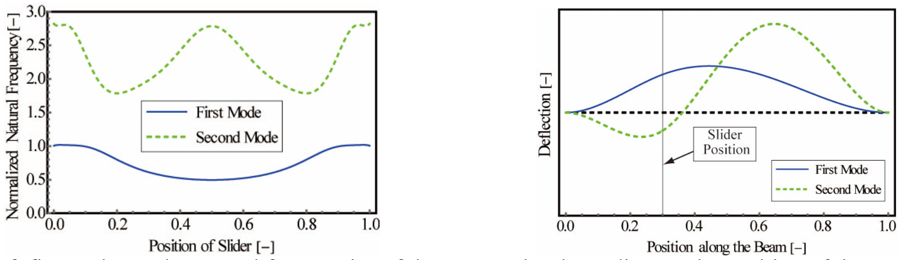

Figure 2: The left figure shows the natural frequencies of the two modes depending on the position of the mass. The right figure shows the corresponding mode shape for a decentered mass.

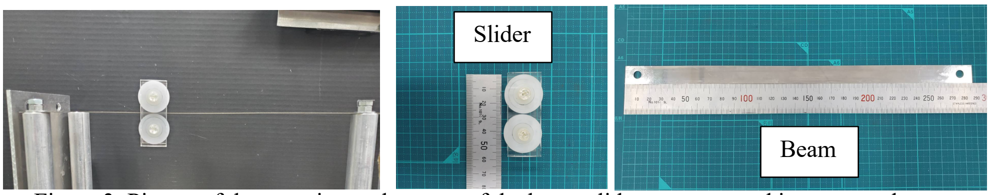

Figure 3: Picture of the experimental system of the beam-slider structure and its separated parts.

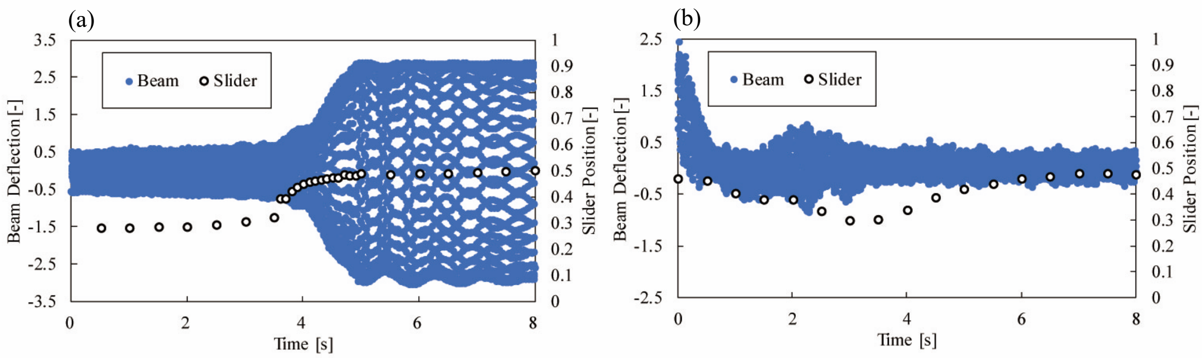

Figure 4: Time histories of slider displacements and beam deflection. The left vertical axis is normalized with the excitation amplitude, while the right vertical axis is normalized with the beam length. The beam deflection was measured at 0.75 according to the right vertical axis. The excitation frequencies were (a)20 Hz and (b)53 Hz while the excitation amplitudes were (a)0.148 mm and (b)0.524 mm.

Experimental Study

Figure 3 shows the parts and the setup of the experimental system. A stainless-steel beam was fixed at both ends to a shaker. In order to make the slider motion as smooth as possible, a pair of plastic bearings were used as the main parts of the slider which interacts with the beam. The gap was small to prevent the slider to rattle or bounce on the beam, but large enough for the slider to move. The slider to beam mass ratio was 1.24, and the first natural frequency of the fixed-fixed beam without the slider was 44 Hz.

Figure 4 shows two of the results obtained by the experiment. (a) is the example of when the system achieved resonance of the first mode. This is caused through the movement of the slider. In the example (b), the slider moved from the center of the beam toward the outer region, then started to move back to the center when it reached a certain position. This motion is difficult to explain when only the second mode with constant mode shape is considered.

Though we have found much more about the beam-slider motion through this experiment, this extended abstract can only provide two examples of the results due to page limitations. Nevertheless, even the two examples alone show that interactions between the vibration modes of the beam are the key to understand the nature of this system.

Conclusion

Experiments were conducted to verify the key aspects to analyze the self-adaptive resonator. The results exhibited the effect of nonlinear modal interactions and therefore theoretical studies ranging to nonlinear analysis of multiple degrees of freedom are essential to truly understand the system.

References

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[3] Thomsen J.J. (1996) Vibration Supression by using Self-arranging Mass. *Journal of Sound and Vibration.* **197**(4):403-425