Analysis of Stable and Unstable Pedestrian Flow Situations in Particle Simulations and Evacuation Experiments

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Summary. Methods to analyze the macroscopic behaviour of systems which are either given as discrete agent-based model or as laboratory experiments are discussed. For demonstration, we address a challenging real-live experiment as application problem within the social sciences: a simplified evacuation scenario is considered where pedestrians have to pass a corridor manoeuvring around a triangular obstacle either left or right. Their decision is influenced by the shortest route to the exit and the route choice of nearby pedestrians. The route choice is investigated under varying the position of the obstacle and by this the length of each route. The macroscopic measure defined as difference of the pedestrian flux left and right from the obstacle shows bistability and a hysteresis behaviour. The branch of unstable flow situations separating the two stable branches where all pedestrians pass the obstacle either left or right was succesfully detected by using a control based continuation. In addition to the analysis of stable and unstable pedestrian flow situations in the pedestrian experiments with humans, a particle model is presented for which the control based continuation algorithm computed the branch of unstable flow situations and reproduced the findings of the pedestrian experiments.

Control-Based Continuation for Laboratory Experiments and Microscopic Simulations

Typically, information from laboratory experiments or microscopic simulations is restricted to ω-limit sets, in the simplest case to stable stationary states. By definition, a system state is moving away from unstable states which can therefore not be directly observed in laboratory experiments or direct simulations. Furthermore, a detailed understanding of the parameter dependent behavior is missing as this would require exhaustive experimental runs or microscopic computations scanning the parameter space. In contrast to this, macroscopic mathematical models successfully formulated for many problems in science and engineering, often allow for a detailed understanding of parameter dependencies of the dynamical behaviour as there are tools available for low-dimensional mathematical models to perform an analytical or numerical bifurcation analysis.

In the present work a method is presented which allows to perform a continuation of stable as well as unstable stationary states in laboratory experiments or microscopic simulations. This approach stabilizes unstable states by adding a non-invasive control which vanishes at the stationary state. The non-invasive control doen't change the stationary state itself but its neighborhood and by this its stability proporties. Often, the dynamic properties of the macroscopic behaviour of a considered laboratory experiment or microscopic simulation system depends on one or several parameters and it is of interest to explore the dynamics depending on such a parameter and to perform a continuation of e.g. stationary states. This approach is called control based continuation [1, 2, 3, 4, 5] and allows to detect also branches of unstable states in addition to the stable branches of the bifurcation diagram.

Suppose the system of interest is represented by

$$\dot{x} = f(x), \tag{1}$$

where the function $f : \mathbb{R} \to \mathbb{R}$ is not explicitly given but the system can be explored by experimental observations measuring the time series $x(t)$.

A non-invasive feedback control is added to the system (1) so that a stationary state of the system with control is also a stationary state of (1) but unstable states are changed to stable ones. This requires that the laboratory experiment and the microscopic simulation is accessible by a control altering the original set-up slightly. The control algorithm used follows a suggestion of [6] and is a feedback control with state observer:

$$\dot{x} = f(x) + a \cdot (y - x) \tag{2}$$

$$\dot{y} = b \cdot (y - x) \tag{3}$$

Equation (2) consists of the original system (1) by adding a control term $a \cdot (y - x)$ which forces for $a > 0$ and a large enough the system to the control target y. The aim is to choose y as unknown unstable stationary state and to stabilize this state with the control. In [2, 3, 4, 5] the unknown target y is computed by using a Newton-method which requires derivative information of suitable quality. The second equation (3) finds this state automatically and for a suitable parameter b unstable stationary states are changed to stable ones.

Varying now a parameter crucial for the system dynamics by changing its value using predictor-corrector methods and adding ideas like pseudo-arclength continuation it is possible to perform a control-based continuation along branches of the bifucation diagram consisting of stable as well as unstable states and also passing saddle-node bifurcation points.

Pedestrian Evacuation Scenario: Real Experiment with Humans and Particle Simulations

The above described approach for control based continuation is demonstrated and tested for a simplified evacuation scenario for pedestrian crowds. In this set-up, pedestrians have to pass a corridor maneuvering around a triangular obstacle

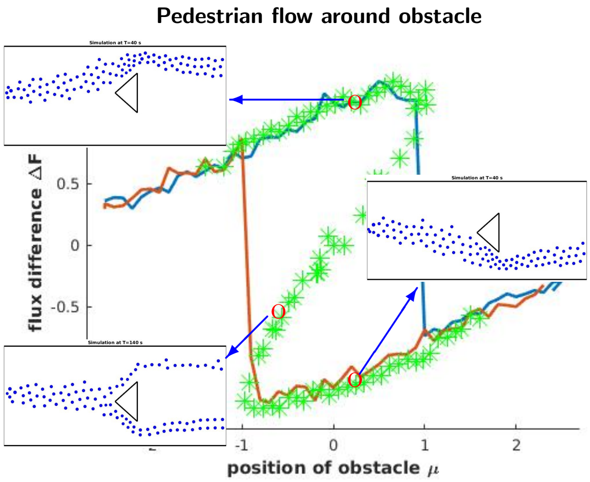

Figure 1: Macroscopic measure of pedestrian flux difference left and right from the obstacle. The insets show the microscopic pedestrian flow situation. The upper and lower branch are stable ones which basins of attraction are separated by an unstable branch. The control-based continuation detected also the unstable branch.

either left or right [7]. The pedestrian's decision is influenced by the shortest route to the exit and the route choice of other pedestrians in a certain neighborhood. The symmetry of the set-up is broken by changing the position of the obstacle and by this the length of each route. The macroscopic measure to be investigated is defined as the difference of the pedestrian flux left and right from the obstacle using the obstacle position as parameter.
As result, the considered pedestrian evacuation scenario shows bistability and a hysteresis behaviour both for the particle model as well as for a real pedestrian experiment with humans. The bifurcation diagram of the particle simulation is shown in Fig. 1. The control-based continuation detects in addition to the two stable branches where all pedestrians pass the obstacle either left or right also an unstable state of the pedestrian flow separating the two stable branches. The insets show the microscopic particle configuation of the observes stable and unstable states of the pedestrian flow.

Conclusion and Outlook

The presented approach is applicable to a wide range of applications in complex systems given by microscopic models or experiments. It extends the gained insight one gets by direct simulations and experiments by adding information about unstable states and by computing the branches of bifurcation diagrams effecively summing up parameter dependent macroscopic system information.

References

[1] Siettos, C. I. ; Kevrekidis, I. G. ; Maroudas, D. (2004). *Coarse bifurcation diagrams via microscopic simulators: A state-feedback control-based approach*. International Journal of Bifurcation and Chaos, **14**(1), 207-220.

[2] Sieber, J., Krauskopf, B. (2008). *Control based bifurcation analysis for experiments*, Nonlinear Dynamics **51**(3), 365–377.

[3] Bureau, E., Schilder, F., Santos, I., Thomsen, J., Starke, J. (2013). *Experimental Bifurcation Analysis of an Impact Oscillator – Tuning A Non-Invasive Control Scheme*. Journal of Sound and Vibration **332**(22), 5883-5897.

[4] Bureau, E., Schilder, F., Elmegård, M., Santos, I., Thomsen, J., Starke, J. (2014). *Experimental Bifurcation Analysis of an Impact Oscillator – Determining Stability*. Journal of Sound and Vibration **333**(21), 5464-5474.

[5] Schilder, F., Bureau, E., Santos, I., Thomsen, J., Starke, J. (2015). *Experimental Bifurcation Analysis – Continuation for Noise-Contaminated Zero Problems*. Journal of Sound and Vibration **358**, 251-266.

[6] Pyragas, K., Pyragas, V., Kiss, I. & Hudson, J. (2002). *Stabilizing and tracking unknown steady states of dynamical systems*. Physical Review Letters **89**, 244103.

[7] Starke, J., Thomsen, K. B., Sørensen, A., Marschler, C., Schilder, F., Dederichs, A., Hjorth, P. (2014). *Nonlinear Effects in Examples of Crowd Evacuation Scenarios*. Pages 560-565. Proceedings of *17th International IEEE Conference on Intelligent Transportation Systems (ITSC 2014)*, Qingdao, China.