Phase - locked breathers in the damped driven granular chains

Margarita Kovaleva[*], Yuli Starosvetsky[**]
[*]*N.N. Semenov Federal Center for Chemical Physics, Russian Academy of Sciences, Moscow, Russia*
[**]*Faculty of Mechanical Engineering, Technion Israel Institute of Technology, Haifa, Israel*

Summary. Over the past few decades, dynamics of one-dimensional (1D) granular lattices has become a subject of immense theoretical and experimental research. In the present talk we will discuss the fundamental problem of nonlinear wave propagation in the damped-driven, granular lattice mounted on a linear elastic foundation which assumes the general type of strongly nonlinear, inter-site potential and subject to an external harmonic forcing in the form of a traveling wave. In the present work we will focus on the analysis of moving breather solution forming in the damped-driven chain.

Introduction

Of late, a special type of localized excitations forming in granular medium has become a subject of immense theoretical and experimental research. This special type of solutions is manifested by a spatial energy localization and time periodicity which are usually referred to in the literature as intrinsically localized modes (ILM) or Discrete Breathers (DB). To the best of authors knowledge the first theoretical study of the dynamics of localized modes in compressed granular chains has been reported in [1]. In fact as it was shown in this study, these localized modes are formed due to the presence of the mass defects. The first experimental study of the formation of long-lived, DB solutions in granular medium has been performed on the compressed, di-atomic granular crystal [2]. This groundbreaking experimental study of DBs has been followed by a systematic theoretical analysis of existence and stability of these spatially localized solutions [3]. All the DB solutions reported in [2-3] correspond to the bright breather solutions emerging in the di-atomic, essentially compressed granular chains. Additional theoretical and experimental study of compressed, mono-atomic granular crystal [4], have demonstrated the existence of dark breather solutions in these chains. These special nonlinear waves have also been reported in theoretical and experimental study of the damped, driven, compressed granular crystals [5]. In fact all the DB solutions existing in granular configurations reported in [2-5] have been considered solely for the pre-compressed state of granular medium. Therefore, when considering the uncompressed state of granular chains, one may wonder whether these spatially localized and time periodic nonlinear wave solutions exist. As a matter of fact, existence of discrete breathers in the un-loaded granular crystals, has been reported at first for one-dimensional, uncompressed granular chain subject to a linear on-site potential and an inter-particle Hertzian interaction [6]. In the same study, formation of static and traveling DBs has been demonstrated numerically. Passing to a small amplitude limit, authors derived the reduced model which has been coined a name of discrete p-Schrodinger (DpS) equation. This model can be regarded as a slow flow model, which approximates the slow (amplitude and phase) modulation of the low amplitude regimes of the original granular setup.

Model

Present study has been motivated by the earlier work by James et al. [8] who derived the analytical approximation for the moving breather supported by the DpS chain. In the present work we consider the DpS chain, subject to the external forcing and dry friction. As it has been shown by James the slow modulation of low amplitude oscillatory solutions can be efficiently described by the discrete p-Schrodinger equation which can be derived through the common multi-scale procedure [6-7]. Following same idea we consider p-Schrodinger equation with forcing and dry friction terms:

$$\frac{d\varphi_k}{dt_1}=\frac{i\sigma}{2}\varphi_k-\lambda\frac{\varphi_k}{|\varphi_k|}+\frac{iF}{2}e^{ik\gamma}+i\mu\left(\varphi_k-\varphi_{k-1}\right)\left|\varphi_k-\varphi_{k-1}\right|^m+i\mu\left(\varphi_k-\varphi_{k+1}\right)\left|\varphi_k-\varphi_{k+1}\right|^m \tag{1}$$

Asymptotic expansion

Assuming the following asymptotic scaling of forcing, damping and the power of non-linearity,

$$m=\nu^2,\ F=\tilde{F}\nu^2,\lambda=\tilde{\lambda}\nu^2,\varphi_n=\left(-1\right)^n\rho_n e^{i\gamma n} \tag{2}$$

we proceed with the multiple-scale expansion,

$$\rho_n=\phi\left(\xi,\tau\right),\ \xi=\nu\left(n-c_q\tau_1\right),\ \tau=\nu^2\tau_1\,. \tag{3}$$

Log – NLS equation

Proceeding to the multiple-scale technique we end up with the damped-driven Log – NLS equation

$$\phi_\tau=-\lambda\frac{\phi}{|\phi|}+\frac{iF}{2}+\frac{3i\beta\cos\gamma}{8}\phi_{\xi\xi}+\frac{3i\beta\Omega^2}{8}\phi\ln|\Omega\phi|,\quad \Omega=\sqrt{\left(2+2\cos\gamma\right)} \tag{4}$$

Seeking for the stationary (in terms of the super-slow time scale) traveling soliton solution we obtain the following second order ODE.

$$\phi_{\xi\xi}+\frac{8i\lambda}{3\beta\cos\gamma}\frac{\phi}{|\phi|}+\frac{8F}{6\beta\cos\gamma}+\frac{\Omega^2}{\cos\gamma}\phi\ln|\Omega\phi|=0 \tag{5}$$

Assuming the phase-locked solution (strictly locked phase $\phi=R(\xi)e^{i\vartheta}$, $R,\vartheta\in\mathrm{Re}$, $\vartheta-const$) the complex ODE equation is split into the real and imaginary part:

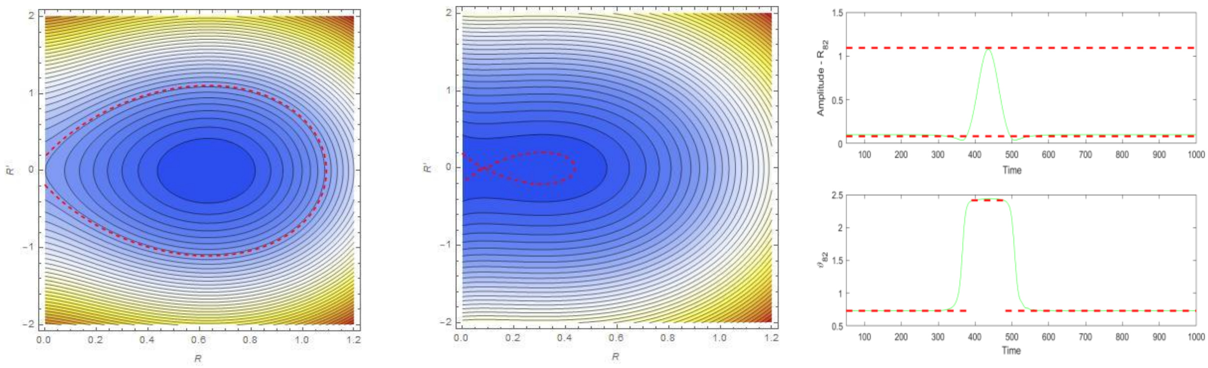
$$R_{\xi\xi}=-\frac{8F}{6\beta\cos\gamma}\cos(\vartheta)-\frac{\Omega^2}{\cos\gamma}R\ln|\Omega R|; \tag{6a}$$

$$-\frac{8\lambda}{3\beta\cos\gamma}+\frac{4F}{3\beta\cos\gamma}\sin\vartheta=0. \tag{6b}$$

The real part is the second order ODE which depicts the evolution of the amplitude, while the second imaginary part is an algebraic equation which defines the stationary phase of the breather solution:

$$\sin\vartheta=\frac{2\lambda}{F};\cos\vartheta=\pm\sqrt{1-\left(\frac{2\lambda}{F}\right)^2} \tag{7}$$

It can be easily inferred from the imaginary part we have two solutions which emerge through the typical saddle – node bifurcation. To illustrate better the dynamics of two distinct breather solutions on both branches we illustrate the following phase portraits.



a) b) c)

Figure 1: Phase planes of the two branches of the equation (a) First branch (b) Second branch (c) Time histories of the response of the DpS chain (Amplitude – upper panel, phase – lower panel), red-dashed lines stand for the phase locked approximation

Clearly the homoclinic orbit of the phase plane shown in the right panel corresponds to the phase locked solitons solution. However, this solution could not be reproduced in the extensive numerical simulations. We conjecture that this solution is unstable. Interestingly enough there exists an additional, phase locked, soliton solution which can be approximated using both phase planes. This solution emanates from the saddle of the right phase plane (Figure 1b) (to the left of the saddle) and continues along the unstable manifold of the saddle up to the point of zero amplitude. When reaching this point there is a jump to the second branch which means that the solution changes phase. Further evolution of the trajectory on the second branch is denoted with the red, dashed line on the left phase plane (Figure 1a). When it reaches again the point of zero amplitude there is a subsequent jump to the first branch and then it gradually converges to a saddle of the first branch along the stable manifold. Here we would like to emphasize that the proposed solution is just an approximation as there is no immediate jump in the phase for the true system solution. This can be clearly seen from the results of Figure 2 where the jump from the vicinity of one phase to the second one, occurs in the fast time scale. Using our analytical model and in particular the analysis of the phase plains, we predict the amplitude, the background and the speed of the traveling soliton. As for the phase of this solution we can clearly see the fast evolution of the phase in the true DpS from one state (first branch) to another (second branch). Obviously enough this transient evolution of the phase from one state to another cannot be captured by our phase locked approximation.

Conclusions

New family of traveling solitons in the forced-damped DpS chain has been observed. The original approach of the analysis of the breathers in the conservative system developed by James [8], allowed us to predict the possibility of formation of phase locked DB in the damped-driven DpS. These solutions can be depicted on the phase plane in the Log-NLS limit.

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