Multifrequency dynamics in an excitable microlaser with delayed optical feedback

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Summary. We investigate theoretically the multi-frequency dynamics observed experimentally in an excitable microlaser subject to delayed optical feedback. We perform a bifurcation analysis of a suitable mathematical model using advanced numerical methods. This demonstrates that the modulated dynamics can be interpreted as quasiperiodic oscillations on a stable torus and hat it originates in the interplay between the delay time and the internal slow timescales of the microlaser.

As sources of short, high-amplitude light pulses, self-pulsing lasers are central to many applications, including telecommunications and neuromorphic photonic computing. We consider a semiconductor micropillar laser with integrated saturable absorber subject to delayed optical feedback [2]. The solitary microlaser is excitable [1]: it is in its non-lasing state, but displays an all-or-none response to external perturbations, in the form of a short, high-amplitude light pulse, depending on whether or not the perturbation amplitude exceeds the so-called excitable threshold [3]. In the presence of a feedback loop, recent theoretical and experimental work demonstrated that a first excitable pulse can regenerate itself when reinjected in the laser after the delay time [5, 4]. As such, a single external perturbation can trigger a train of pulses whose repetition rate is close to the delay time. It has recently been shown [4] that several pulse trains can be triggered and sustained simultaneously: in such a case, a very slow convergence is typically observed towards a pulsing pattern where the pulses are equidistant in the laser cavity. In other words, the system is slowly attracted towards one of the stable pulsing periodic solutions with periods close to submultiples of the delay time.
Here, we show theoretically and experimentally that the delayed feedback can also induce complex multifrequency dynamics. In particular, we provide experimental evidence of the existence of pulsing regimes with a strong modulation of the pulse amplitude on a slow timescale compared to the delay time.

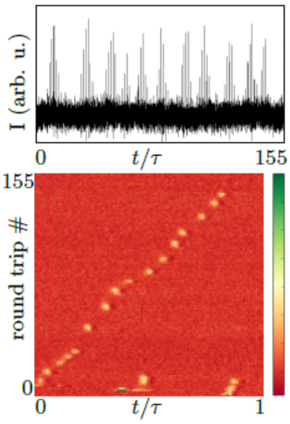

Figure 1: Experimental intensity time series, represented over reduced time (top) and in the pseudo-space (bottom) where the x-axis represents one delay time and the y-axis represents the number of roundtrips in the laser external (feedback) cavity.

We focus on the theoretical investigation of such multifrequency dynamics and perform a numerical bifurcation analysis of the Yamada model with feedback. This system of three delay-differential equations (DDEs) for the gain G, the absorption Q and the intensity I is written as follows in the dimensionless form:

$$\begin{aligned}\dot{G} &= \gamma_G(A - G - GI);\\ \dot{Q} &= \gamma_Q(B - Q - aQI);\\ \dot{I} &= (G - Q - 1)I + \kappa I(t-\tau).\end{aligned} \tag{1}$$

Here, A is the pump parameter, B is the non-saturable absorption, a is the saturation parameter and γ_G and γ_Q are the carrier recombination rates in the gain and absorber media, respectively. The values of γ_G and γ_Q are typically small, and the model (1) is then a slow-fast dynamical system with two slow variables (the gain G and the absorption Q) and one fast variable (the laser field intensity I). The optical feedback is described by the delayed term in the intensity equation, where κ is the feedback strength and τ is the feedback delay. This model has been shown to describe accurately the dynamics observed experimentally [6, 4].

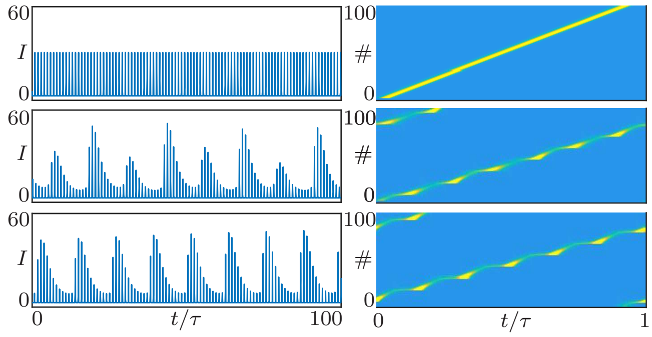

Figure 2: Simulated intensity time series represented over reduced time (left) and in the pseudo-space (right), for κ = 0.39 (top), κ = 0.3805 (middle), and κ = 0.38 (bottom). The delay is τ= 965.

Time domain simulations of the Yamada model with feedback, in figure 2, demonstrates its ability to reproduce the modulated pulsing dynamics observed in the experiment. We perform an in-depth bifurcation analysis of the model (1) with advanced numerical methods [7], considering both the feedback strength κ and feedback delay τ as bifurcation parameters. It demonstrates that the modulated dynamics can be interpreted as quasiperiodic oscillations on a stable torus. The bifurcation analysis unveils an interplay between the delay time and the internal timescales of the microlaser (related to the carrier recombination rates in the gain and absorber sections) as the main reason behind the emergence of such multifrequency dynamics. Moreover, time-domain simulations of the Yamada model with delay show that chaotic dynamics exists in small regions of the parameter space.

Overall, our results provide a better understanding of pulsing dynamics in an excitable laser with delayed feedback. As such, they constitute a step toward an all-optical control of pulse trains, which may prove useful for optical memories or neuromorphic photonic computing. Despite its simplicity, the Yamada model with feedback describes accurately the complex dynamics observed in the experiment and displays complex dynamics, including chaotic behaviour. Importantly, the only ingredients, excitability and feedback, are very general [1]. Therefore, we believe our results may be of interest beyond the scope of laser dynamics.

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