# Bistability in nonlinear elastic robotic arms subject to delayed feedback control

Giuseppe Habib*, Asier Barrios‡, Andras Bartfai*, Zoltan Dombovari*
*MTA-BME Lendület Machine Tool Vibrations Research Group, Department of Applied Mechanics, Budapest University of Technology and Economics, Budapest, Hungary
‡Dynamics and Control Department, Ideko, Elgoibar, Spain

*Summary*. Stability and bifurcation analysis of a non-rigid robotic arm controlled with a time delayed feedback loop is addressed in this work. The study aims at revealing the dynamical mechanisms leading to the appearance of limit cycle oscillations existing in the stable region of the trivial solution of the system, which are related to the combined dynamics of the robot control and its structural nonlinearities. An analytical study of the bifurcations occurring at the loss of stability enables the development of strategies to eliminate this undesired bistable phenomenon by the implementation of special additional nonlinearities in the control force.

## Introduction

Robots are increasingly adopted in modern manufacturing facilities, thanks to their versatility and relatively low cost [1]. Milling operation is one of the operations robots are intended to be used for, where complicated tool trajectories can be realized in a large workspace, with a relatively low cost. The relative vibrations between workpiece and tool are a troublesome phenomenon in milling that is mainly caused by the so-called regenerative vibration. The main solution to avoid them is to increase stiffness and damping and try to disturb time delays introduced by the regenerative effect [2]. Generally, increasing the stiffness is hardly achievable, since robotic arms are naturally slender and not especially stiff structures [3]. This makes them particularly prone to vibrations. The main method to mitigate these vibrations should consists of implementing an active controller working collaboratively in the robot original feedback loop. This envisioned controller online reads the acceleration of the end effector (EE, see Fig. 1a) as input and sends a proportional signal to the robot controller in order to counteract and suppress the arising oscillations. This signal is combined to the signal of the position controller of the robot required to make the robotic arm follow the prescribed path during machining.
Although this procedure is rather straightforward to be implemented, there are several aspects that can undermine the effectivity of such combined system if the followings are not properly accounted for: (i) Robotic arms are naturally slender and they cannot be assumed to be rigid, especially if they are subject to strong periodic forces, as in the case of machining. (ii) Since the actuators are placed at the joints of the arm, the system is underactuated. Depending on the position of the sensors, either near the motor or near the EE, the system can be considered as collocated or non-collocated, which have relevant consequences on the system stability [4, 5]. (iii) Robot configuration changes continuously during operation and the drive components of the robot generate non-negligible nonlinearities; as we will illustrate in this study, these nonlinearities might have important consequences on the system robustness. (iv) Robot's controller is unavoidably subject to time delay in the feedback loop. Although this is often negligible, if large control gains are required to counteract strong forces, time delay can still generate instabilities.
This study is motivated by the appearance of unexpected vibrations in a real industrial robotic arm in milling operation. This robot is equipped with a built-in controller (most probably a proportional-derivative controller) for its correct positioning and with an additional controller proportional to the EE acceleration, to counteract machining vibrations (Fig. 1a). Although the control parameters of the system were set such that the system was stable (Fig. 1b), when subject to very small external forcing, in some occasions the robotic arm exhibited either large or small oscillations, which suggests that it was in bistable conditions (see Fig. 1c). The objective of this work is to define and study a general simplified model of this system in order to understand the origin of the bistability and define methods to avoid it. From a broader prospective, this seed research aim at providing a reliable modelling of robotic manufacturing.

## Mathematical model

The mathematical model adopted is a two degrees of freedom (DoF) system (Fig. 1a), consisting of two lumped masses $m_1$ and $m_2$, connected by a linear damper $c$, and a nonlinear spring $k_{\rm nl}$. The two DoF of the system represent the two dominant DoFs measured for the actual robotic arm in a certain frequency bandwidth. The nonlinearity models a stiffness nonlinearity observed during measurement, most probably originated in the joints. This simplified equivalent mechanical model captures the most important features of the real robotic arm considered in the study. The model is equivalent since during a simple dynamic measurement on the EE the source of the stiffness, damping and nonlinearities are hard to be traced. A prescribed reference trajectory $x_{\rm d}$ is programmed, such that, in ideal circumstances, an identical constrained motion $x_{\rm r}$ is imposed to $m_1$ via a spring of stiffness $k$ with a certain time delay $\tau_r$. This enables the robot to follow the prescribed path. The equations of motion has the following form:

$$\begin{aligned} &m_1\ddot{x}_1 + c\,(\dot{x}_1-\dot{x}_2) + k_{\rm nl}(\Delta x)\,(x_1-x_2) + kx_1 = kx_{\rm r}, \\ &m_2\ddot{x}_2 + c\,(\dot{x}_2-\dot{x}_1) + k_{\rm nl}(\Delta x)\,(x_2-x_1) = 0, \end{aligned} \tag{1}$$

where $k_{\rm nl}(\Delta x) = k_2 + \kappa\Delta x^2$ ($\Delta x := x_2 - x_1$). Apart from the position controller integrated in the robot, an additional signal $x_{\rm f}$, proportional to the acceleration of the EE, is added to $x_{\rm r}$. This generates a final constrained motion $x_{\rm r}(t) =$

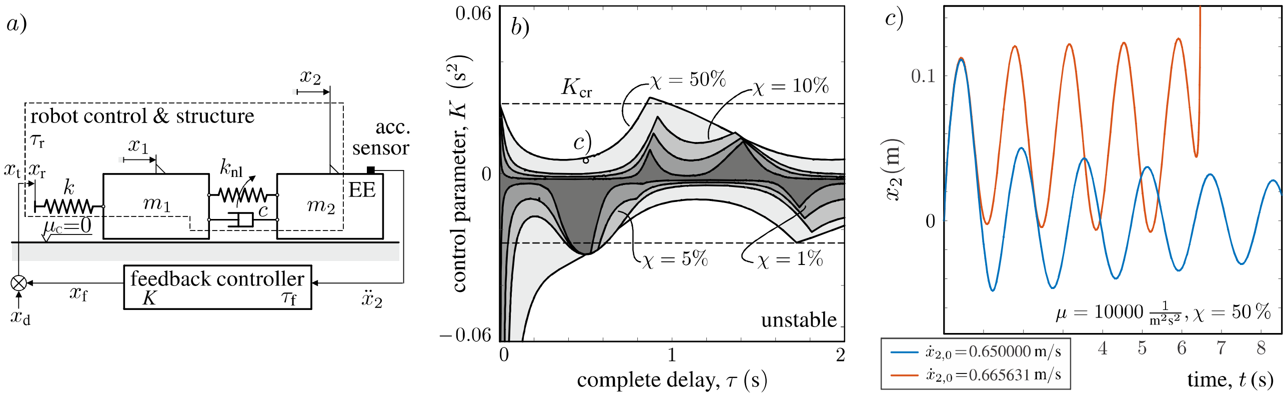


Figure 1: a) shows the sketch of the 2 DoF model with the additional acceleration feedback control ; b) stability chart in the $(\tau, K)$ space for $\omega_1 = \omega_2 = 2\pi$ rad/s, $r = 1$, $\chi =: (50,\ 10,\ 5,\ 1)\,\%$, $K_{\rm cr} = 0.0253\,{\rm s}^2$, c) time evolutions for different initial conditions.

$x_{\rm d}(t - \tau_{\rm r}) + K\ddot{x}_2(t - \tau_{\rm r} - \tau_{\rm f})$, where $\tau_{\rm f}$ is the delay of the acceleration feedback. In order to focus on the instabilities generated by the acceleration feedback control, in this work we assume constant desired position, that is, $x_{\rm d}(t) := x_{\rm d}$, which results in the equilibrium $(\overline{x}_1, \overline{x}_2) = (x_{\rm d}, x_{\rm d})$. By introducing the perturbations $x_1 := \overline{x}_1 + u_1$ and $x_2 := \overline{x}_2 + u_2$, the stability of the local equilibrium can be studied. Via a standard non-dimensionalization procedure, the equations of motion around the equilibrium are reduced to the equation, where $r := m_2/m_1$, $\omega_1^2 := k_1/m_1$, $\omega_2^2 := k_2/m_2$, $\chi := c/(2m_2\omega_2)$, $\mu := \kappa/m_2$, $\ddot{u}_{2\tau} := \ddot{u}_2(t - \tau)$ and $\tau := \tau_{\rm r} + \tau_{\rm f}$.

$$\begin{aligned}
&\ddot{u}_1 + 2\chi r\omega_2\left(\dot{u}_1 - \dot{u}_2\right) + \omega_2^2 r\left(u_1 - u_2\right) + \mu r(u_1 - u_2)^3 + \omega_1^2 u_1 = \omega_1^2 K\ddot{u}_{2\tau},\\
&\ddot{u}_2 + 2\chi\omega_2\left(\dot{u}_2 - \dot{u}_1\right) + \omega_2^2\left(u_2 - u_1\right) + \mu(u_2 - u_1)^3 = 0,
\end{aligned} \tag{2}$$

## Stability and bistable behavior

By linearising (2) and setting $\chi := 0$, the linear stability of the corresponding neutral equation can be investigated (see Fig. 1b). A naive necessary condition of stability for neutral equation is to have the neutral coefficient (here $\omega_1^2 K$) less than one in its magnitude with $|K_{\rm cr}| \le \omega_1^{-2}$. However, the investigated robotic arm is non-collocated; therefore, the neutral condition is always fulfilled, thus, the stability domains overtakes $K_{\rm cr}$ . In the case of small damping, repeating lobe structure significantly erodes the stable region, which is limited to a narrow region around $K = 0$.
Apart from the local stability of the trivial solution, time simulations show that the nonlinearity of the system has significant impact on its global stability. In particular, if the system is subject to a stiffening nonlinearity ($\mu > 0$) simulations for parameter values within the stable region tend to diverge if initial conditions are sufficiently large. This phenomenon is caused by the subcritical characteristic of the bifurcations occurring at the stability limit and it is probably directly related to the bistable behaviour observed in real robotic arms. The detailed analysis of the bifurcation behaviour of the system enables us to design additional nonlinearities to be included in the control force algorithm to enforce supercritical behaviour; therefore, eliminating the bistable behavior in the stable region.

## Conclusions

The stability and bifurcation analysis of a simplified model of the robotic arm subject to acceleration feedback was performed. Results illustrated that the stability chart is characterized by a critical value of the control gain, which is a necessary condition to guarantee stability, and by a repeating stability limit pattern, which strongly depends on the time delay and on system damping. The mechanism connecting bistable behavior and hardening nonlinearity was also identified. The full understanding of this mechanism enables the development of a control algorithm, based on nonlinear functions, which forces the bifurcation to be supercritical, suppressing bistable behaviour in the stable region.

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