Nonlinear Dynamics of a Ring-Type MEMS Gyroscope

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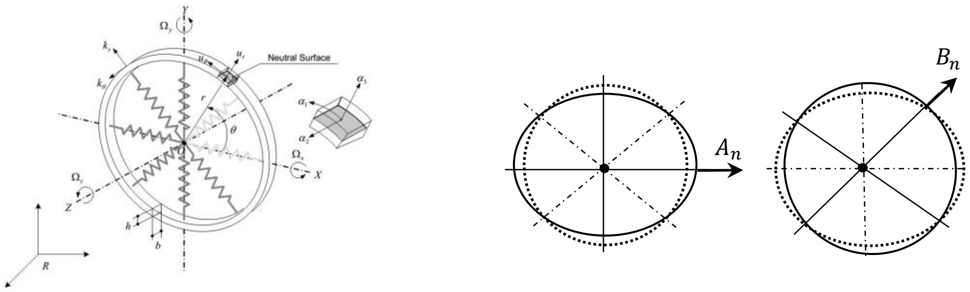
Summary. This paper investigates the nonlinear dynamic behavior of a rotating ring that forms an essential element in MEMS ring-based vibratory gyroscopes that utilize oscillatory nonlinear electrostatic forces. Mathematical model that incorporates geometric nonlinearities for the MEMS ring structure as well as a model that represents nonlinear electrostatic forces that act on the ring structure are formulated. Galerkin's procedure is employed to reduce the equations of motion to a set of ordinary differential equations. Understanding the effects of nonlinear actuator dynamics is considered important for characterizing the dynamic behavior of such devices. For investigating the dynamic response behavior of a ring-type vibratory angular rate, the equations of motion are simplified by ignoring the extensional vibrations, since the second resonant flexural mode is excited in this class of applications. Dynamic responses in the driving and the sensing directions are examined via time responses, phase diagram, and Poincare' map plots when the input angular motion and the nonlinear electrostatic force are considered simultaneously. The analysis is envisaged to aid fabrication of this class of devices as well as for providing design improvements in MEMS-based Ring-type Gyroscopes.

Concept and Modeling

In the present paper, nonlinear dynamic behavior of rotating thin circular rings for use in vibratory angular rate sensors have been investigated via numerical simulations. A homogenous, isotropic ring is chosen for the angular rate sensor. Cho [1] developed mathematical models for rotating ring based angular rate sensors for the purpose of investigating linear as well as nonlinear dynamic behavior and dynamic stability of angular rate sensors which are subjected to external excitation. In the past, Huang and Soedel [2] and Natasiavas [3], among others, have also studied the nonlinear dynamic behaviour of rotating thin circular rings. Dynamic response behavior of rotating thin circular rings for use in vibratory angular rate sensors was investigated by Gebrel et al [4] via numerical simulations by employing the linearized model considering the second mode. In this study, a suitable theoretical model is developed for the purposes of representing the nonlinear electromagnetic actuation forces that are used for exciting the ring from two positions to obtain improved device sensitivity. Figure 1(a) illustrates the geometry and parameters used in the present study while Figure 1(b) illustrates the two degenerate modal configurations associated with the second flexural mode, and are separated by 45 degrees. The schematic of the rotating ring geometry used in present study have been described in detail in [1, 4].



(a) (b)

Figure 1: (a) schematic of the rotating ring geometry, and (b) Visualization of second flexural modes of ring

In the present paper, the nonlinear equations of motion in terms of the generalized coordinates associated with the flexural coordinates A_n and B_n [1, 5] are developed for the purposes of illustrating the dynamic behavior. The discretized equations of motion that govern the dynamics with suitable nonlinear harmonic electrostatic forces are derived:

$$\left[\rho h\pi + 2\rho h\pi\left(\frac{n\gamma}{2r}\right)^2 A_n^2\right]\ddot{A}_n + 2\rho h\pi\left(\frac{n\gamma}{2r}\right)^2 A_nB_n\ddot{B}_n + 2\zeta\omega_0\dot{A}_n + \left[\frac{EI}{br^4}(n^2-1)n^2 + \rho h\Omega^2(n^2-2) + k_r\right]\pi A_n +$$

$$\left[\frac{EA}{br^2} + k_r\right]\left(\frac{n\gamma}{2r}\right)^2[A_n^2 + B_n^2]\pi A_n + 2\rho h\pi\left(\frac{n\gamma}{2r}\right)^2[\dot{A}_n^2 + \dot{B}_n^2]A_n - \rho h\pi\dot{\Omega}\frac{1}{n}B_n - 2\rho h\pi\Omega\frac{1}{n}\dot{B}_n = f_{Nes}(A_n, B_n, \theta_i)cos(\omega t) \qquad (1)$$

$$\left[\rho h\pi + 2\rho h\pi\left(\frac{n\gamma}{2r}\right)^2 B_n^2\right]\ddot{B}_n + 2\rho h\pi\left(\frac{n\gamma}{2r}\right)^2 A_nB_n\ddot{A}_n + 2\zeta\omega_0\dot{B}_n + \left[\frac{EI}{br^4}(n^2-1)n^2 + \rho h\Omega^2(n^2-2) + k_r\right]\pi B_n +$$

$$\left[\frac{EA}{br^2} + k_r\right]\left(\frac{n\gamma}{2r}\right)^2[A_n^2 + B_n^2]\pi B_n + 2\rho h\pi\left(\frac{n\gamma}{2r}\right)^2[\dot{A}_n^2 + \dot{B}_n^2]B_n + \rho h\pi\dot{\Omega}\frac{1}{n}A_n + 2\rho h\pi\Omega\frac{1}{n}\dot{A}_n = 0 \qquad (2)$$

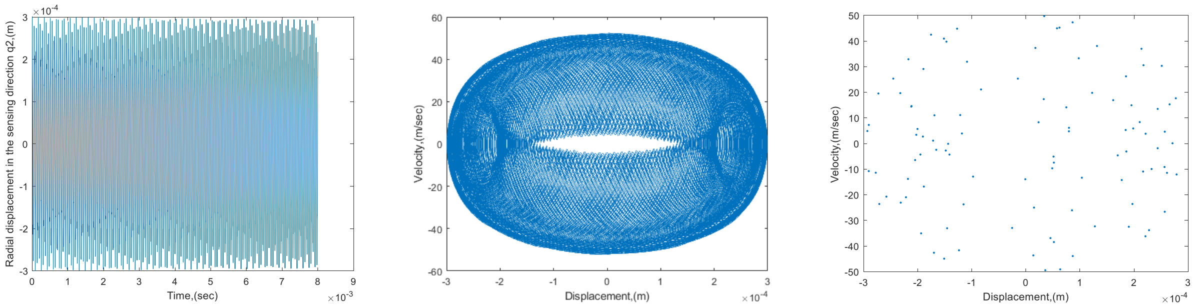
where ρ is mass density, EI represents flexural rigidity, A is the cross sectional area of ring, b denotes axial thickness of ring, h is radial thickness, r is the mean radius of the ring, and k_r support spring stiffness in the radial direction. The quantification of the nonlinear terms are governed by the parameter γ. Oscillatory external nonlinear electrostatic actuator force that acts along the A_n direction is considered to have a magnitude f_{Nes} and frequency ω , while input angular velocity and acceleration, respectively, are denoted by Ω and $\dot{\Omega}$. The area moment of inertia of the ring cross section about its neutral axis is expressed as $I = bh^3/12$. The parameter ζ represents the mechanical damping ratio, and n denotes the number of modes which is taken to be 2 in the present study. The angular positions of electrostatic forces that excite the ring in the primary direction is denoted by θ_i , $i = 1,2,3,4$. In order to represent the oscillatory electrostatic force, a suitable theoretical formulation is employed:

$$f_{Nes} = \sum_{i=1}^{4}(\cos(n\theta_i) - \left(\frac{n\gamma}{2r}\right) A_n) * \left[\frac{\varepsilon_0 V^2 a}{2\left\{d - A_n \cos(n\theta_i) - B_n \sin(n\theta_i) + \frac{n\gamma}{4R}[A_n^2 + B_n^2]\right\}^2}\right] \quad , \tag{3}$$

where the parameter ε_0 represents the permittivity of air, V represents the applied voltage between the electrode and the ring, a represent the electrode width which represents the space between electrode and the surface of the ring, d is the distance between electrode and ring

Results and Discussion

For the purposes of predicting the nonlinear response characteristic of MEMS ring-type gyroscope, equations (1) and (2) have been solved numerically. Parameters associated with a typical MEMS ring-type gyroscope are considered. The following ring design parameters: radius of 500 μm, thickness of 12.5 μm , and a height of 30 μm with Young's modulus of 210 Gpa and the density of 8800 Kg/m^3 have been chosen. At a nominal input angular rate of $2\pi\ rad/sec$ and a typical device high quality factor of 1×10^5 , the resulting frequencies have been evaluated as $\omega_1 = 2.1422 \times 10^5\ (rad/sec)$, and $\omega_2 = 2.1428 \times 10^5\ (rad/sec)$. The generalized coordinates $q_1 = A_n/h, q_2 = B_n/h$ have been used for the non-dimensional equations. For an input angular velocity $\Omega = 2\pi\ (rad/sec\ )$, under nonlinear oscillatory electrostatic excitation, the time response of the ring in the sensing direction in the presence of geometric nonlinear terms is depicted in Figure 2(a). Figure 2(b) depict the phase portrait based on the steady-state response in the sensing direction. The effects of nonlinearities due to the nonlinearities of the system as well as nonlinear electrostatic force are evident from the plots. Furthermore, nonlinearity can be seen in the Poincare' map results as shown in Figure 2(c), where the Poincare' maps appear as a cloud of unorganized points in the phase plane in Figure 2(b) due to the influence of nonlinear terms in the model as well as nonlinear actuator. Internal resonance behavior is not analyzed in this study since the natural frequencies are close to each other and cannot be equal in the typical device operating range.



(a) (b) (c)

Figure 2: (a) Time Response , (b) Phase diagram, and (c) Poincare' map.

Conclusions

Nonlinear dynamic behavior of a MEMS-scale ring-type vibratory gyroscope has been examined via numerical simulations. The device exhibits nonlinearity in the presence of geometric nonlinear term in the model which may be attributed to high vibration amplitudes. In addition, nonlinearities due to electrostatic actuation have also been incorporated. Results on the dynamic response obtained via time-response, Phase portraits and Poincare' maps indicate significant influence of geometric nonlinearities on the resulting steady state behavior. The analysis is envisaged to aid fabrication of this class of devices as well as for providing design improvements in MEMS Ring-based Gyroscopes.

References

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