Internal Resonances in Magnetic Resonance Force Microscopy

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Summary. We investigate the existence of internal resonances in magnetic resonance force microscopy asymptotically and numerically and demonstrate the possible existence of energy transfer from a directly excited mode to its out-of-plane counterpart. The reconstitution multiple-scales method reveals the existence of non-stationary dynamics that yields coexisting periodic and quasiperiodic dynamics for small damping that may enable multi-functional detection of both electron spin and the sample magnetic properties.

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Magnetic resonance force microscopy (MRFM) is an imaging technique that enables acquisition of three-dimensional magnetic images at nanometer scales, and has been adapted for detection of magnetic spin of a single electron [1]. It is based on combining the technologies of magnetic resonance imaging (MRI) with atomic force microscopy (AFM). In conventional MRI devices the electronic spins are detected by measuring their magnetic induction using an inductive coil as an antenna. However, in MRFM the detection is implemented mechanically using a cantilever to directly detect a modulated spin gradient force between the sample spins and a ferromagnetic particle attached to the tip of the cantilever. While MRFM systems are receiving a growing amount of interest, to date, a comprehensive theoretical treatment is still lacking. Existing models are based on simplistic lumped-mass reductions that include linear estimates of cantilever stiffness and damping complemented by a nonlinear approximation of the magnetic force [2] and are unable to resolve the spatio-temporal complexity of the magneto-elastic sensor.

We thus consistently formulate a nonlinear initial-boundary-value problem (IBVP) combining the three-dimensional motion of a viscoelastic micro-cantilever and the dynamic interactions of the spin magnetic moments (see Figure 1-left). The MRFM cantilever IBVP incorporates the generalizes forces defined by the total magnetic field which are augmented by the time-dependent spin magnetic moment components in a rotating system of coordinates described by the Bloch equations [3]. We reduce the IBVP to a seventh-order nonlinear dynamical system and investigate the three-dimensional motion of the MRFM cantilever tip corresponding to adiabatic and non-adiabatic conditions. We emphasize that periodic base excitation of the vertical MRFM configuration here is not sensitive to a global homoclinic escape bifurcation threshold [4] typical of the traditional horizontal configuration of the cantilever sensor which is limited to operation below a jump-to-contact condition.

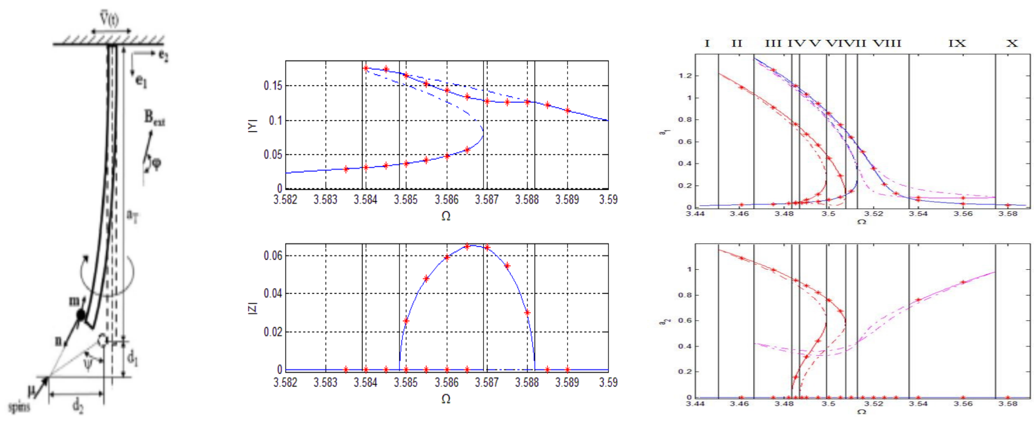

Figure 1 – MRFM model definition sketch (left), frequency response for the case of a 1:1 internal resonance (center) and for the case of a 2:1 internal resonance (right).

We use an asymptotic reconstitution multiple-scale analysis to accurately determine the MRFM system frequency response which enables estimation of the cantilever frequency shift corresponding to documented measurements. We investigate the stability of slowly varying evolutions for the conditions of both one-to-one (Figure 1-enter) and two-to-one internal resonances (Figure 1 – right) which reveal the existence coexisting periodic solutions and secondary Hopf bifurcations [5].

Numerical integration of the dynamical system for different values of system parameters near its two-to-one internal resonance reveal symmetry breaking of a fundamental period-doubled solution (Figure 2) which evolves to an asymmetric period three ultra-sub-harmonic and culminates with non-stationary solutions depicted by a dense power spectra and corresponding poincare' map**.**

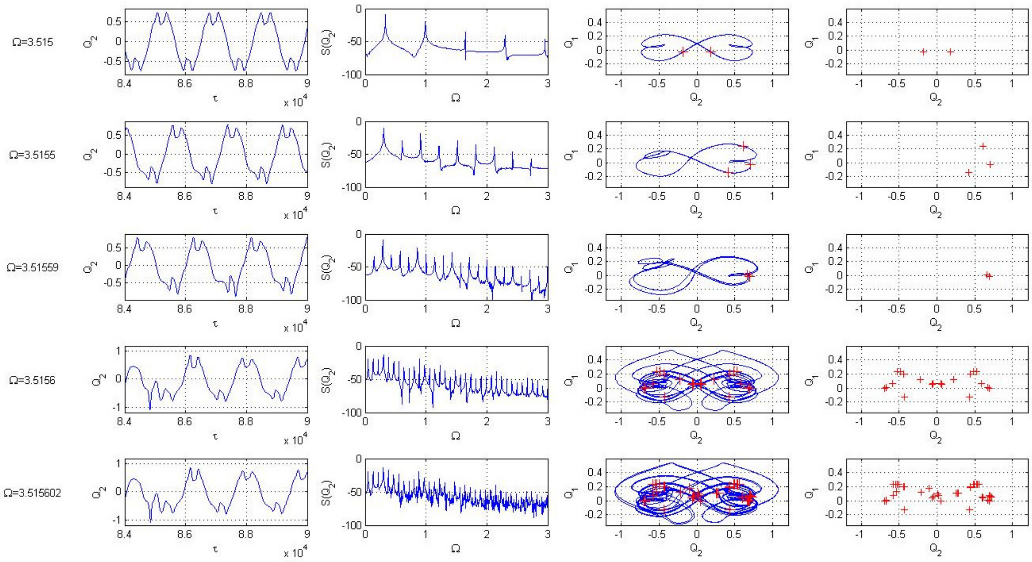

Figure 2 – Numerical response for a 2:1 internal resonance which exhibits complex periodic ultra-sub-harmonic solutions.

Numerical integration of the dynamical system for different values of system parameters near its one-to-one internal resonance (Figure 3) reveal quasiperiodic and chaotic like motion which in addition to identification of the spin gradient force may enable simultaneous multi-functional sensing of material properties of magnetized samples.

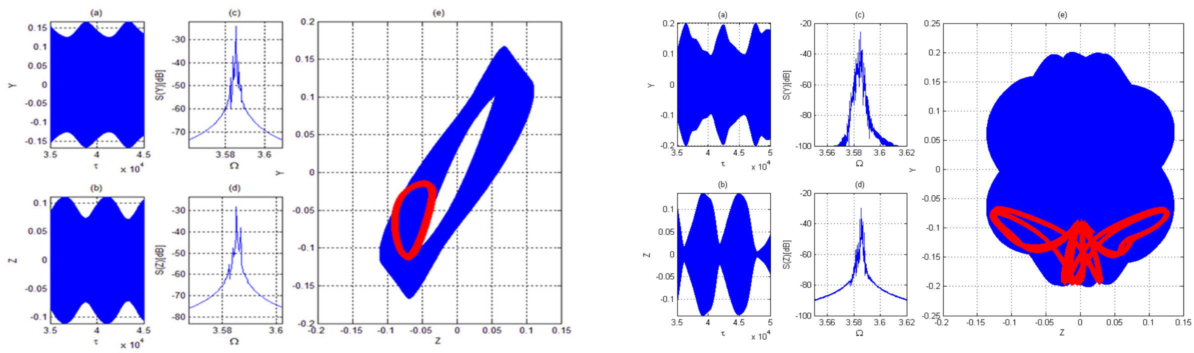

Figure 3 – Numerical response for a 1:1 internal resonance which exhibits a quasiperiodic torus (left) and a strange attractor (right).

References

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