Controlling the location of discrete breather formation in a nonlinear electrical lattice using random excitation

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Summary. Emergence of *discrete breathers* (also known as Intrinsic Localized Modes) in nonlinear lattices, analogous to *solitons* in continuous media, is a uniquely nonlinear phenomenon wherein energy gets localized in specific nodes of the lattice. While extensively studied in various contexts including optical systems and MEMS arrays, the effects of random excitation on the formation and dynamics of breathers remains largely unexplored. In this talk, we will present our recent results which demonstrate that additive white noise excitation can aid the *controlled* formation and sustenance of discrete breathers in a nonlinear electrical lattice. In addition, we will present computational results that demonstrate that a temporary burst of random excitation of the white noise type can facilitate the change of location of a breather in a lattice. The results are expected to be of both theoretical interest as well as in several important applications involving discrete breathers, such as targeted energy transfer.

Overview and Analytical Framework

Discrete breathers (also known as Intrinsic Localized Modes) correspond to localization of energy at specific locations within perfectly periodic, nonlinear lattices. Manifest as sustained, stable oscillatory modes, the existence of breathers is predicated on the nonlinearity and discreteness of the lattice. Studies of discrete breathers may be traced back to investigations of the Fermi-Pasta-Ulam oscillator chain. Since then, the phenomenon has been observed in a broad spectrum of systems such as Josephson junctions, photonic crystals [2] and micro-cantilever (MEMS) arrays [3] thereby establishing its universality. In addition to being fundamentally important in theoretical studies of energy distribution in periodic nonlinear systems, breathers are of interest in applications such as targeted energy transfer and signal processing.
The dynamical scale of several systems in which breathers emerge point to the importance of stochastic effects. Furthermore, it is known that phenomena such as stochastic resonance that arise due the interplay between randomness and nonlinearity can be exploited to advantage in applications. However, the influence of randomness on the formation and dynamics of breathers is yet to be understood and motivates the present effort. In recent work, we found that random excitation (characterized by additive white noise of appropriate intensity) of even an arbitrarily chosen cell of a lattice for a short period of time can induce breather formation in a nonlinear electrical lattice [1]. Here, we specifically ask whether random excitation of a node at which a breather has emerged can induce a change of location of the breather. In other words, we ask whether random excitation can be used as a technique to manipulate and move breathers to different locations in a lattice.

Analytical Framework

The non-dimensional dynamic model equations for a unit cell “n" in the considered electrical lattice are given in Equations (1) and (2) [4]:

$$\frac{dv_n}{d\tau} = \frac{1}{c(v_n)}\Bigg[y_n - i_D(v_n) \\ -\frac{1}{\omega_0 C_0}\left(\frac{1}{R_I}+\frac{1}{R}\right)v_n + \frac{cos(\Omega\tau)}{RC_0\omega_0}\Bigg] \tag{1}$$

$$\frac{dy_n}{d\tau} = \frac{L_2}{L_1}(v_{n+1}+v_{n-1}-2v_n) - v_n \tag{2}$$

In these equations, $v_n = V_n/V_s$ is the non-dimensional node-voltage of cell n, $\tau = \omega_0 t$ is the non-dimensional time, $\Omega = \omega/\omega_0$ is the non-dimensional driving frequency with $\omega_0 = 1/\sqrt{L_2C_0}$, $c(v_n) = C(V_n)/C_0$, and $i_D(v_n) = I_D(V_n)/w_0C_0V_s$. Note that the sinusoidal sources in each cell are identical in frequency Ω, amplitude V_s, and phase. R_l is an additional term added to account for the resistance of the inductors. It is considered to be parallel to L_2 and its value is dependent on the driving voltage amplitude V_s. In this case, where we have chosen $V_s = 4V$, R_l is equal to $5\ k\Omega$ [**?**].
To account for stochastic excitation, the modelling equations are recast as a vector-valued Ito stochastic differential equation given in Equation (3):

$$d\mathbf{x}(\tau) = f(\mathbf{x}(\tau),\tau)d\tau + \sigma\tilde{G}d\mathbf{W}(\tau) \tag{3}$$

where $d\mathbf{W}(t)$ is a Brownian motion increment that is drawn from the standard normal distribution and represents a white-noise Gaussian process. The noise intensity is represented by the scalar coefficient σ. The computational results are obtained by simulating the above equations using the standard Euler-Maruyama scheme for numerical solutions of stochastic differential equations.

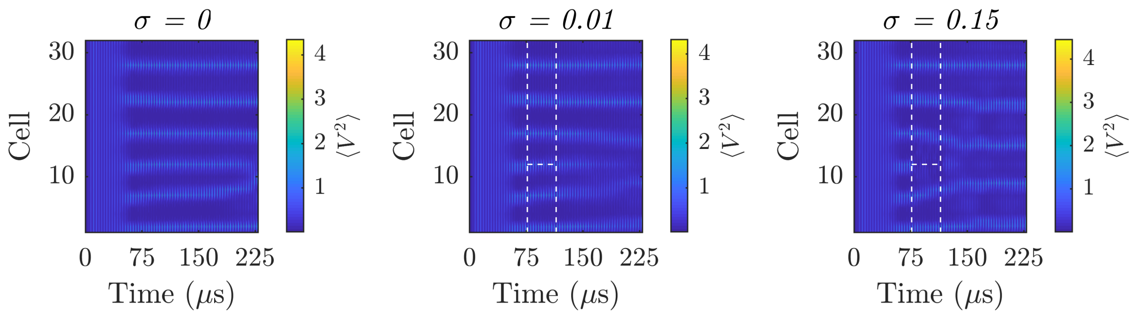

Figure 1: Averaged Results for Noise Burst at Cell 12

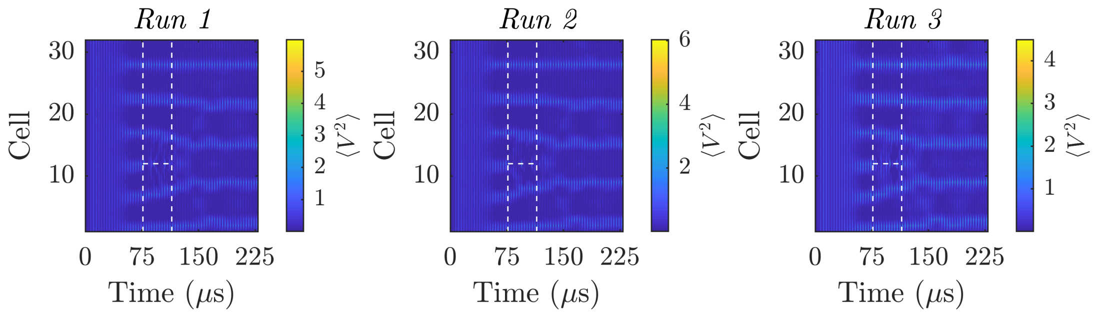

Figure 2: Individual Run Results for Noise Burst at Cell 12

Results

Breather formation spurred by small randomized initial voltage conditions at top node of cell in the range of 0 to 0.04 volts is observed. White noise is applied at cell 12 starting at 1/3 into the simulation, turned off again at the halfway point into the simulation. Figure 1 depicts the average of ten individual runs each using the same initial conditions but different noise profiles, with the dashed white lines indicating when and where noise is applied. As apparent, when $\sigma = 0.15$, the breather located at cell 12 is effectively eliminated. Individual run results pictured in Figure 2 further corroborate this. The ability to eliminate or shift breathers was found to depend on the initial condition profile used. For 2 of the 3 profiles used, including this one, it was possible to move or eliminate the breathers using a noise burst at its location. For the third profile tested, the breathers were too robust and reappeared after the noise burst in the same location. However, the effectiveness in the other two IC cases indicate that noise may be used to control the location of these breathers on average. The other cases suggest that appropriate choice of noise intensity is essential to successfully move the breathers.

Conclusions

The results indicate that random excitation characterized by additive white noise can induce a change of location of breathers in the lattice. This has not been reported in the literature and hence is a novel aspect with implications in applications where shepherding breathers across a lattice would be of interest. In addition, our recent results showing that discrete breather formation in a nonlinear electrical lattice may be controlled by random excitation will also be presented in the talk. Together, the results suggest that random excitation can be employed as a novel pathway towards controlling the emergence and dynamics of discrete breathers in nonlinear lattices.

References

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