# Parametrically driven morphing of thin piezoelectric surfaces

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*Summary*. The possibility of exploiting the parametric resonance phenomena to perform morphing of piezoelectric surfaces is here investigated analytically and numerically. The case study consists of a PVDF beam subject to an in-plane pulsating strain applied through voltage variation. The dynamical excitation induces an out-of-plane parametric resonance of the beam which can be driven to excite desired individual modes or combination of them through nonlinear coupling. A nonlinear reduced order model for a piezoelectric thin Euler-Bernoulli beam is developed considering the multi-physics piezo-elastic coupling. The conservation of the electric charge is enforced in 3D while the equations of motion are expressed in 1D using the arclength parametrization along the beam centerline. The analytical treatment is based on the method of multiple scales and allows to obtain the region of the forcing parameters for which the parametric resonance is achieved. The analytical solutions are validated against numerical results provided by the finite element code ABAQUS through which a full 3D nonlinear model is addressed. The analytically obtained transition curves (representing the boundary between resonant and non resonant behavior in the space of forcing parameters) and the frequency response curves are compared to those obtained numerically achieving a good agreement. The voltage thresholds for which the parametric resonances are induced, and the robustness of the responses suggest that the investigated phenomenon is a promising strategy for surface dynamic morphing.

## Introduction

A piezoelectric formulation valid rigorously only for parallelepiped-shaped beams is proposed. The model is focused on the parametric resonances induced by a pulsating voltage whose gradient is defined along the thickness [1]. The transition curves defining the forcing parameters for which the beam exhibits a parametric resonance are obtained using the method of multiple scales [2, 3]. The nonlinear equations of motion of the continuous piezoelectric beam are reduced employing a full-basis Galerkin-discretization of the continuous piezoelectric beam [4, 5].
Finally, the closed form asymptotic results are compared with those obtained by the direct integration of the nonlinear dynamic problem performed with the nonlinear finite element software ABAQUS [6]. The results show a good agreement and the simplified hypotheses adopted for the piezoelectric beam model are confirmed. The implemented assumptions could be also used for developing a simplified piezoelectric shell model.

## Equations of motion

Consider the 3D Euclidean space with a Cartesian fixed frame $(s, y, z)$ where the position of each point can be defined by a vector $\boldsymbol{x} = s\boldsymbol{e}_1 + y\boldsymbol{e}_2 + z\boldsymbol{e}_3$. The straight beam exhibits a rectangular cross-section of edges $b$ and $h$ and the position of each section along the beam span $l$, in the reference configuration, is described the vector $\boldsymbol{r}^0 = s\boldsymbol{e}_3$ with $s \in [0, l]$. The local frame in the reference configuration is denoted by $(\boldsymbol{b}_1{}^0, \boldsymbol{b}_2{}^0, \boldsymbol{b}_3{}^0)$. Unit vectors $\boldsymbol{b}_1{}^0$ is collinear with $\boldsymbol{e}_1$ while the pair $(\boldsymbol{b}_2{}^0, \boldsymbol{b}_3{}^0)$ represents a centered and principal frame for the beams cross-section.
The mechanical problem is formulated in the plane $\boldsymbol{e}_1 - \boldsymbol{e}_3$, and the displacement of the central line is denoted by $\boldsymbol{u}(s) = u(s)\boldsymbol{b}_1{}^0 + v(s)\boldsymbol{b}_3{}^0$. The rotation of the beam cross sections is $\boldsymbol{\theta}(s) = \theta(s)\boldsymbol{b}_2{}^0$ where $\boldsymbol{b}_2{}^0$ coincides with $e_2$. According to Saint-Venant ansatz, stresses and strain states are simplified: $\sigma_{22} = \sigma_{33} = \sigma_{23} = \sigma_{12} = 0$ $\varepsilon_{22} = \varepsilon_{33} = \gamma_{12} = \gamma_{23} = \gamma_{13} = 0$.
On the other hand, it is assumed that the voltage $V$ varies only in the $z$ direction according to the electric boundary conditions to which it is subject. We are interested in the problem in which the top beam surface at $z = h/2$ is connected to ground and presents $V(h/2) = 0$. On the bottom surface a voltage different from zero is assigned, $V(-h/2) = \overline{\Phi}$. Moreover, it is assumed that the potential varies with a quadratic law across the relatively small beam thickness according to

$$V(z, t) = \Phi_0(t) + \Phi_1(t)z + \Phi_2(t)\frac{z^2}{2}. \tag{1}$$

. Imposing the potential boundary conditions given above yields

$$\Phi_1(t) = -\frac{\overline{\Phi}(t)}{h}. \tag{2}$$

The beam axial force and the bending moment can be obtained by integration over the cross-section of the elastic axial tension.
The equations of motion for the nonlinear beam in the current local frame can be written as

$$N' + \frac{\mu}{\nu}M' = \rho A\ddot{u}\cos\theta + \ddot{v}\sin\theta, \tag{3}$$

$$\mu N + (\frac{M'}{\nu})' = -\rho A\ddot{u}\sin\theta + \ddot{v}\cos\theta. \tag{4}$$

where the shear force has been condensed using equation for the balance of the angular momentum, the prime indicates differentiation with respect to $s$ and the overdot with respect to time $t$. Equation (4) can be condensed considering that the axial force is constant along the beam.

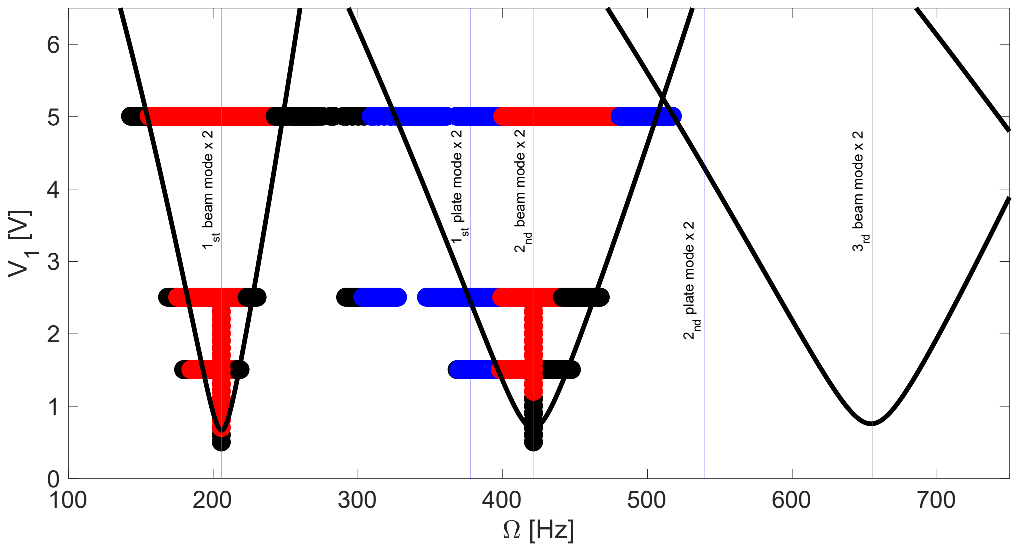


Figure 1: Parametric resonance regions for the PVDF sample where the black solid lines are obtained according to the fifth order solutions for the lowest three beam modes assuming $\zeta = 3$ % and $V_0 = 5$ V; the black and red circles denote nonresonant and resonant responses provided by ABAQUS while the blue circles indicate resonant responses for the plates modes; the gray and blue vertical thin lines are placed at twice the frequencies of the beam-like modes and twice the plate-like frequencies, respectively; the black dashed horizontal lines denote the amplitudes of the AC voltage $V_1$ for which the ABAQUS simulations were run, namely, $(1.5, 2.5, 5)$ V.

## The Method of Multiple Scales

The unknown displacement is expressed as follows

$$v(s,t) = \epsilon v_1(s,t_0,t_2,t_4) + \epsilon^3 v_3(s,t_0,t_2,t_4) + \epsilon^5 v_5(s,t_0,t_2,t_4). \tag{5}$$

and $\Phi_1 = V_0 + \epsilon^2 V_1 \cos \Omega t$ where the forcing frequency is $\Omega = 2\omega + \epsilon^2 \sigma$. The parameter $\sigma$ represents the detuning of the forcing frequency with respect to twice of the frequency of the excited mode. Moreover, the damping ratio is assumed as $\epsilon^2 \zeta$. Collecting the terms with the same order of $\epsilon$, the nonlinear equation of motion can be written up to the 5th order and solved asymptotically.

## Results

The transition zones identified by ABAQUS are in accordance with the transition curves evaluated with the closed form expression. In fact, the threshold points provided by ABAQUS are very close to the asymptotically obtained curves (see Fig. 1). In particular, the correction obtained with the 5th order frequency response curves (black solid lines) show that the fully numerical and reduced model are in close agreement.

## Conclusions

The present paper investigates the parametric resonance conditions induced by a pulsating voltage in a PVDF copolymer beam/plate. The piezoelectric properties of the PVDF film are exploited to induce an initial tensile stress that makes the structure stiffer and increases the natural frequencies. A nonlinear piezoelectric beam model is developed and the transition curves for the lowest three modes are computed using the method of multiple scales. The obtained results are validated via ABAQUS through which the piezoelectric system is addressed within geometrically nonlinear problem formulation.

## References


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