Application of proportional "ghost" damping for sliding or yielding structures in time history dynamic analyses

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Extended abstract

When studying the response of a structure to a dynamic excitation, the modelling of energy dissipation has always been a challenge to the engineers. Different methodologies were set up for different kinds of problems. These include the modelling of friction forces, hysteretic behavior and restitution coefficients for non-linear structures. For linear structures, modelled with finite elements, the general practice is to use either viscous equivalent modal damping or viscous proportional "Rayleigh" damping.

Content overview

The first part of the paper gives a quick overview of the most usual representations of energy dissipation in dynamic analyses, with a particular focus on the use of a proportional Rayleigh damping and its drawbacks when applied to a linear but not linearly supported structure. The second part describes the "ghost" damping methodology set in place to overcome those drawbacks. The third part presents tests case basis on which the "ghost" damping was checked, including simple examples representative of a handling crane, a fuel rack and a seismically isolated building.

PART 1: Representation of energy dissipation in usual engineering practices

Values for modal damping are found in numerous seismic design codes around the world and they constitute the energy dissipation modelling basis for most applications in the industry (see references [1], [2], [3],and [5]). It has become common practice to try to apply these modal damping values to dynamic problems that are not solved on a modal basis. To do so with a finite element model, the most widely applied method consists in building a viscous damping matrix [C] as a linear combination of the mass matrix [M] and the stiffness matrix [K], α and β being referred to as the Rayleigh coefficients. This method is referred to as the Rayleigh method, after the work of Baron Rayleigh in the 1870s (see [6] in the references). It is possible to create a [C] matrix that produces a required given modal damping value, although for only two structural modes at most.

For the case of linear structures, with a non linear supporting system, which will be the main focus of this paper, an unwanted effect of using a Rayleigh damping, even only on the linear part of the model, is that it produces damping forces that will spuriously limit, and ultimately stop the motion of the structure relative to its support basis. Indeed, the α[M] part of the Rayleigh damping matrix is predominantly diagonal. These diagonal terms, when multiplied by the velocity vector, produce damping forces proportional to the velocity of each node relative to the calculation referential, as illustrated by equation (1) for a two mass system, with a single degree of freedom for each node (U_X is the displacement of node X relative to the calculation referential, M_X the mass attached to this node).

$$\left\{F_{\alpha\_damping}\right\} = \alpha \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \begin{Bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{Bmatrix} = \begin{Bmatrix} \alpha M_1 \dot{U}_1 \\ \alpha M_2 \dot{U}_2 \end{Bmatrix} \qquad (1)$$

Any rigid body motion of this two mass system relative to the calculation referential gives rise to non intended resisting damping forces. This phenomenon will be referred to as the "spurious damping of rigid body motions" in this paper.

On the other hand, the β[K] part of the Rayleigh damping produces forces that are only proportional to the relative velocities between nodes connected by an element. As an example, if a spring of stiffness K_1 connects the two masses of the simple system described earlier, the resulting damping forces are illustrated by equation (2).

$$\left\{F_{\beta\_damping}\right\} = \beta \begin{bmatrix} K_1 & -K_1 \\ -K_1 & K_1 \end{bmatrix} \begin{Bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{Bmatrix} = \begin{Bmatrix} \beta K_1 (\dot{U}_1 - \dot{U}_2) \\ \beta K_1 (\dot{U}_2 - \dot{U}_1) \end{Bmatrix} \qquad (2)$$

These forces do represent dissipation of energy because of structural deformation, which is generally the objective assigned to Rayleigh damping.

The inadequacy of using the α[M] part of the Rayleigh damping is known and some references can be found in regulatory documentations such as [1] and [2]. As a consequence, cautious engineers will only use the β[K] part and therefore loose the ability to properly damp the lower frequency modes of the structures. Such approach invariably results in overly conservative estimations of the structural responses.

PART 2: Description of the 'GHOST" damping methodology

The so-called "ghost" damping methodology has been specifically developed for the case of linear structures on a non linear support. Its objective is to overcome the two drawbacks of the Rayleigh proportional damping method identified in Part 1: controlled modal damping value on only two modes and spurious damping of rigid body motions. In the case of non linearly supported linear systems, the non linear support is generally explicitly modelled and is itself a source of energy dissipation, through friction coefficients (case of a handling crane) or modelling of a hysteretic material behaviour (case of a seismically isolated building). The linear part is modelled by linear stiffness and mass matrices, [K] and [M], which remain constant throughout the calculations.

The "ghost" methodology aims at producing damping forces on the linear structure which are only proportional to the structure deformation velocities. This is what happens with the usual Rayleigh damping for purely linear systems. To achieve this goal, the rigid body motion of the linear part of the model is subtracted from its overall movement when constructing the damping forces vectors. The form of these desired damping forces is illustrated in equation (3) for the two-masses-one-stiffness system described earlier.

$$\left\{F_{\alpha\_damping}\right\} = \begin{Bmatrix} \alpha M_1 (\dot{U}_1 - \dot{U}_{g1}) \\ \alpha M_2 (\dot{U}_2 - \dot{U}_{g2}) \end{Bmatrix} \text{ and } \left\{F_{\beta\_damping}\right\} = \begin{Bmatrix} \beta K_1 (\dot{U}_1 - \dot{U}_2) \\ \beta K_1 (\dot{U}_2 - \dot{U}_1) \end{Bmatrix} \tag{3}$$

The β[K] part is of the same nature as the proportional Rayleigh damping β[K] matrix. The α [M] part results in forces proportional to the node velocity minus the rigid body motion velocity at the node location. In this equation, U_{gX} represents the displacement of node X due only to the rigid body motion.

PART 3: Tests cases basis for the "GHOST" damping methodology

Tests cases will be developed in the full article.

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Conclusions

Two drawbacks of using a classic proportional Rayleigh damping methodology in the dynamic analysis of non-linearly supported linear structures were highlighted: spurious damping of rigid body motions and under-damping of some major modes located in between the first and the last mode frequencies of interest. These drawbacks are known and the cautious engineer will usually make conservative assumptions to avoid the first one. The present paper describes a methodology to suppress the spurious damping of rigid body motion by using a "ghost" structure within a FE model and generate damping forces proportional to the actual structure velocities relative to its "ghost". At the same time, different Rayleigh coefficients are used for different directions, in order to bring the equivalent modal damping values of the linear part of the model closer to the targets in each direction, often defined in the codes.

References

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