Membrane flutter induced by radiation of surface gravity waves on a uniform flow

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Summary. We consider stability of an elastic membrane being on the bottom of a uniform horizontal flow of an inviscid and incompressible fluid of finite depth with free surface. The membrane is simply supported at the leading and the trailing edges which attach it to the two parts of the horizontal rigid floor. The membrane has an infinite span in the direction perpendicular to the direction of the flow and a finite width in the direction of the flow. For the membrane of infinite width we derive a full dispersion relation that is valid for arbitrary depth of the fluid layer and find conditions for the flutter of the membrane due to emission of surface gravity waves. We describe this radiation-induced instability by means of the perturbation theory of the roots of the dispersion relation and the concept of negative energy waves and study its relation to the anomalous Doppler effect.

Membrane flutter as a radiation-induced instability

Flutter of membranes is a classical subject for at least seven decades. Membranes submerged in a compressible gas flow and their flutter at supersonic speeds have been considered already in the works [1, 2]. Recent works on the membrane flutter are motivated by such diverse applications as stability of membrane roofs in civil engineering [3], flutter of traveling paper webs [4] and aerodynamics of sails and membrane wings of natural flyers [5, 6].

Surface gravity waves on a motionless fluid of finite depth is a classical subject as well, going back to the seminal studies of Russell and Kelvin [7]. Numerous generalizations are known taking into account, for instance, a uniform or a shear flow and surface tension [8], flexible bottom or a flexible plate resting on a free surface [9]. The latter setting has a straightforward motivation in dynamics of sea ice and a less obvious application in analogue gravity experiments [7].

Remarkably, another phenomenon that is being analysed from the analogue gravity perspective is super-radiance [7] and its particular form, discovered by Ginzburg and Frank [10], known as the anomalous Doppler effect (ADE) [11, 12]. In electrodynamics, the ADE manifests itself when an electrically neutral overall particle, endowed with an internal structure, becomes excited and emits a photon during its uniform but superluminal motion through a medium, even if it started the motion in its ground state; the energy source is the bulk motion of the particle [11].

Anomalous Doppler effect in hydrodynamics was demonstrated for a mechanical oscillator with one degree of freedom, moving parallel to the boundary between two incompressible fluids of different densities [13]. It was shown that the oscillator becomes excited due to radiation of internal gravity waves if it moves sufficiently fast. In [14] the ADE for such an oscillator was demonstrated due to radiation of surface gravity waves in a layer of an incompressible fluid.

Nemtsov [15] was the first who considered flutter of an elastic membrane being on the bottom of a uniform horizontal flow of an inviscid and incompressible fluid as an anomalous Doppler effect due to emission of long surface gravity waves. In the shallow water approximation, he investigated both the case of a membrane that spreads infinitely far in both horizontal directions and the case when the width of the membrane in the direction of the flow is finite whereas the span in the perpendicular direction is infinite. Nevertheless, the case of the flow of arbitrary depth has not been studied in [15] as well as no numerical computation supporting the asymptotical results has been performed. Another issue that has not been addressed in [15] is the relation of stability domains for the membrane of the finite width to that for the membrane of the infinite width.

Vedeneev studied flutter of an elastic plate of finite and infinite widths on the bottom of a uniform horizontal flow of a compressible gas occupying the upper semi-space. He performed analysis of the relation of stability conditions for the finite plate with that for the infinite plate using the method of global stability analysis by Kulikovskii [16, 17]. However, no connection has been made to the ADE and the concept of negative energy waves.

In the present work we reconsider the setting of Nemtsov in order to address the finite height of the fluid layer, find flutter domains in the parameter space, analyze them using perturbation of multiple roots of the dispersion relation, find the domains of absolute and convective instability and investigate dependence of the flutter onset on the width of the membrane including the limit of infinite width. We will explain the instabilities via the interaction of positive and negative energy waves by finding an appropriate G-Hamiltonian formulation and relate them to the anomalous Doppler effect.

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