Stochastic resonance in a parametrically perturbed aeroelastic system

Varun H. S.*, M. S. Aswathy* and Sunetra Sarkar*
Department of Aerospace Engineering, Indian Institute of Technology Madras, Chennai, India

Summary. In this work, we investigate the effect of parametric noise on a classical two degree-of-freedom pitch-plunge aeroelastic system and study the manifestation of stochastic resonance in the same. The non-dimensional form of the governing equations are studied, considering nonlinear soft-springs. The reduced velocity of the flow is modelled as a stochastically varying parameter by the Ornstein-Uhlenbeck (OU) process. This parametric noise significantly changes the qualitative dynamics of the system. One such new qualitative dynamics that we observe is noise induced intermittency, where the system hops between two attractors– $\vec{0}$ fixed point and a stable limit cycle oscillation (LCO). Next, we fix the mean reduced velocity near the onset of bifurcation and examine effect of noise intensity. We see that the signal to noise ratio (SNR) of the system responses reaches a maxima for an optimum value of the noise intensity. This characteristic feature in the aeroelastic system perturbed by parametric noise indicates the phenomenon of stochastic resonance.

Introduction

The response of an elastic structure in a fluid flow is a very important research area as the applications range from aeroelastic system design to design of tall buildings, bridges [1]. The design of these structures requires careful consideration as there is interplay between three kinds of forces — inertial, elastic and aerodynamic forces. The interaction between the three forces can cause the structure to exhibit LCOs, which is undesirable as it can cause fatigue failure in the structure [2]. These self-excited LCOs of the structure is termed as the flutter phenomena. Recently, a lot of importance is being given to the role played by parametric noise in such engineering systems [3, 4]. Noise is known to bring drastic qualitative changes in the dynamical behaviour of such systems. One important feature in physical and biological systems subjected to noise is the phenomenon of stochastic resonance [9, 10], which manifests due to a change in noise intensity. It is known that such systems attain a maximum SNR at an optimum noise intensity. Inspired from these works, we make an attempt to study the effect of parametric noise and the role played by the noise intensity on a classical pitch-plunge aeroelastic system.

Methodology

The aeroelastic system is modelled as an airfoil undergoing motion in the pitch and plunge degrees-of-freedom under a steady, uniform incoming flow and is based on the model by Lee *et al* [6, 7]. The non-dimensional governing equations of motion are given in Equation 1.

$$\epsilon'' + x_\alpha \alpha'' + 2\zeta_\epsilon \left(\frac{\bar{\omega}}{U}\right)\epsilon' + \left(\frac{\bar{\omega}}{U}\right)^2 (\epsilon + \beta_\epsilon \epsilon^3) = -\left(\frac{1}{\pi\mu}\right) C_L(\tau)$$
$$\left(\frac{x_\alpha}{r_\alpha^2}\right)\epsilon'' + \alpha'' + 2\left(\frac{\zeta_\alpha}{U}\right)\alpha' + \left(\frac{1}{U^2}\right)^2 (\alpha + \beta_\alpha \alpha^3) = \left(\frac{2}{\pi\mu r_\alpha^2}\right) C_M(\tau) \tag{1}$$

where $x_\alpha b$ is the distance between the elastic axis and the centre of mass of the airfoil, $r_\alpha b$ is the radius of gyration of the airfoil, b is the semi-chord length of the airfoil. $'$ denotes differentiation with respect to non-dimensional time, ϵ is the non-dimensional plunge of the elastic axis, α is the pitch of the elastic axis, U is the reduced velocity, $\bar{\omega}$ is the ratio of uncoupled natural frequencies in the plunge to the pitch mode, μ is the mass ratio, β_α and β_ϵ denote the coefficients of the cubic term of stiffness in the pitch and plunge modes respectively, ζ_α and ζ_ϵ are the damping ratios in the pitch and plunge modes respectively. C_L and C_M are the lift and moment coefficients respectively, which are derived based on the Wagner function formulation [5]. The springs are assumed to be soft springs (β_α is $-ve$).

Noise model

The reduced velocity U in Equation 1 is stochastically modelled as an OU process, which is generated by the Stochastic Differential Equation (SDE) given in Equation 2

$$dU = \lambda(U_m - U)\,dt + \sigma\,dW \tag{2}$$

where U_m is the mean reduced velocity, λ is the inverse of the correlation time, σ is the noise intensity – parameters of the OU process, W represents the standard brownian motion. The generated process has a correlation $R_{UU}(\Delta\tau) = \exp(-\lambda\Delta\tau)$ with variance $\sigma^2/(2\lambda)$. The entire system when cast in state space form looks like Equation 3

$$d\vec{X} = \mathbf{f}(\vec{X}, U, \tau)\,d\tau$$
$$dU = \lambda(U_m - U)\,d\tau + \sigma\,dW \tag{3}$$

where $\vec{X}$ consists of the system and auxillary variables (see [6, 7]). The Equations in 3 are interpreted as an Itô SDE and are integrated by using the Euler-Maruyama method [8, 9].

Results and Discussions

Firstly, the bifurcation behaviour of the deterministic system is studied by varying reduced velocity (U). The system undergoes a sub-critical Hopf bifurcation (flutter point, $U = 6.29$) beyond which it gets attracted to a stable LCO. The unstable LCO takes a turn and becomes the stable LCO branch (turning point, $U = 5.93$). Now we investigate how these responses get altered in the presence of parametric noise. Figure 1 (a) and (b) show time histories of $U(\tau)$, $\alpha(\tau)$ respectively for $\sigma = 0.37$ and Figure 1 (c) and (d) shows the same for $\sigma = 0.27$. For $U_m = 6.25$ the system starts to display hopping dynamics (Figure 1 (b) and (d)), wherein it intermittently switches between the $\vec{0}$ fixed point and the LCO. This effect is solely due to the presence of parametric noise in the system and is dubbed as noise induced intermittency in the literature [3, 11]. Next, we fix $U_m = 6.25$ and vary the noise intensity σ. As σ is increased, the SNR value of

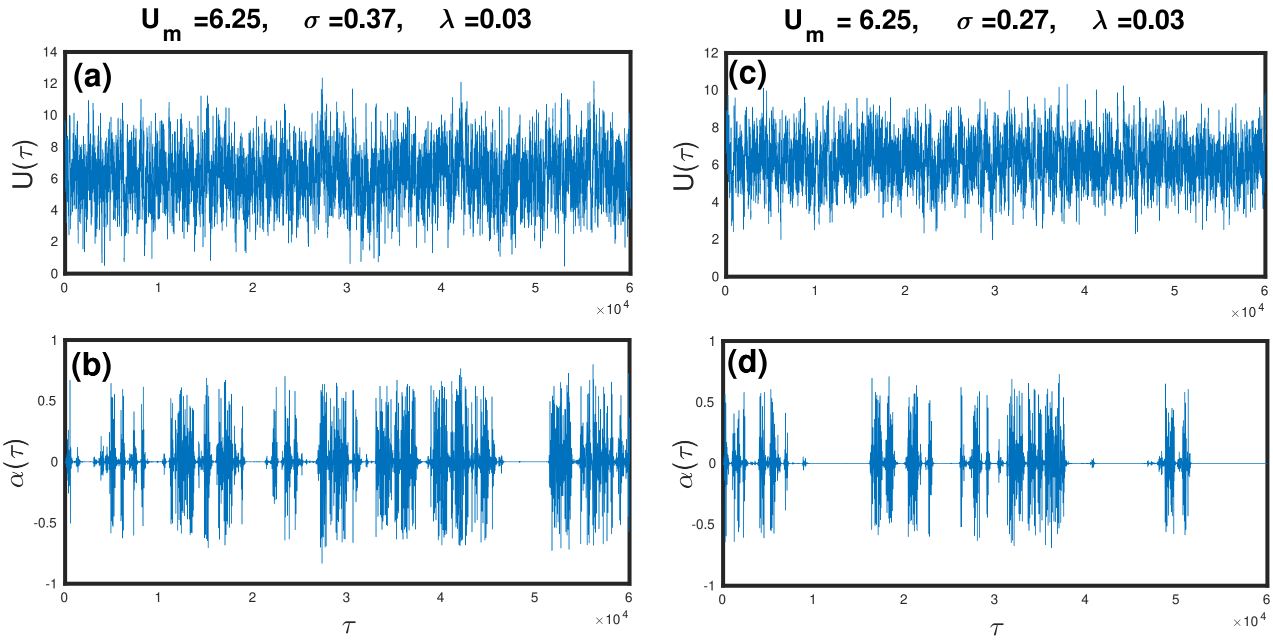

Figure 1: Time histories for $\alpha(\tau)$ and $U(\tau)$. (a) and (b): $\sigma = 0.37$, (c) and (d): $\sigma = 0.27$

the response increases as the system starts spending greater amounts of time in the LCO attractor. Further increase in σ leads to a decrease in the value of SNR due to frequent switching between the attractors and the dynamics gets dictated by the noise. This is further confirmed by plotting the power spectra and mean residence times of the responses. This phenomenon where the SNR reaches a maxima for an optimum value of intensity of the noise is termed as stochastic resonance [9, 10] and is being reported in the aeroelastic system for the first time.

Conclusions

We have investigated the effects of parametric noise on the considered aeroelastic system. The introduction of parametric noise in U, modelled as an OU process brings about drastic changes in the system dynamics. The system starts hopping between the two attractors and displays a new state of intermittent oscillations. The phenomenon of stochastic resonance is observed when the noise intensity σ is varied. These changes in the system dynamics and the manifestation of stochastic resonance brought on due to the parametric noise presents new challenges during design and use of aeroelastic systems.

References

[1] Hodges D. H., Pierce G. A. (2011) Introduction to Structural Dynamics and Aeroelasticity. Cambridge University Press .

[2] Schijve J. (2009) Fatigue damage in aircraft structures, not wanted, but tolerated? *Int. J. Fatigue* **31**:998-1011.

[3] Venkatramani J., Krishna S. K., Sarkar S., Gupta S. (2017) Physical mechanism of intermittency route to aeroelastic flutter. *J. Fluids Struct* **75**:9-26.

[4] Aswathy M. S., Sarkar S. (2019) Effect of stochastic parametric noise on vortex induced vibrations. *Int. J. Mech Sci* **153-154**:103-118.

[5] Fung Y. C. (1955) An Introduction to the theory of Aeroelasticity. John Wiley and Sons, New York.

[6] Lee B. H. K., Gong L., Wong Y.S. (1997) Analysis and Computation of Nonlinear Dynamic Response of a Two-Degree-of-Freedom System and its Application in Aeroelasticity. *J. Fluids Struct* **11**:225-246.

[7] Lee B.H.K., Jiang L.Y., Wong Y.S. (1999) Flutter of an Airfoil with Cubic Restoring Force. *J. Fluids Struct* **13**:75-101.

[8] Higham D.J. (2001) An Algorithmic Introduction to Numerical Simulation of Stochastic Differential Equations. *SIAM Rev.* **43**:525-546.

[9] Rajasekar S., Sanjuan M.A.F. (2016) Nonlinear Resonances. Springer International

[10] Dykman M., McClintock P. (1999) Stochastic Resonance. *Sci Prog* **82**:113-134.

[11] Krishna S. K., Sarkar S., Gupta S. (2019) Multiplicative noise induced intermittency in maps. *Int. J. Nonlin Mech* **117**:103251