Non-smooth dynamics modeling of drill-string systems in heterogeneous formations

Arviandy G. Aribowo*, Roeland Wildemans*, Emmanuel Detournay† and Nathan van de Wouw *,†
*Department of Mechanical Engineering, Eindhoven University of Technology, The Netherlands
†Department of Civil, Environmental and Geo-Engineering, University of Minnesota, U.S.A.

Summary. An extension of the drill-string dynamics model for drilling in heterogeneous rock formations is presented to study the heterogeneity effect on both the axial and torsional dynamics.

Introduction

According to [1], the total loads acting on the Polycrystalline-Diamond-Compact (PDC) bit in rotary drilling systems (see Figure 1-left) are changed abruptly when the bit blades drill into interchangeably harder and softer layers of heterogeneous rock formations. This fast load-change condition due to the formation heterogeneity can affect the dynamic response of the system and therewith the vibrational signature at the bit, which in turn may influence the durability of the PDC bit and drilling efficiency. To this end in this study, the drill-string system model as presented in [2, 3] is extended for vertical drilling scenario in interbedded formations, particularly for the transitional part of the motion of the bit in two heterogeneous layers (see Figure 1-middle). The latter requires a novel bit-rock interaction model of the forces and torques acting on the bit in such transitional phase.

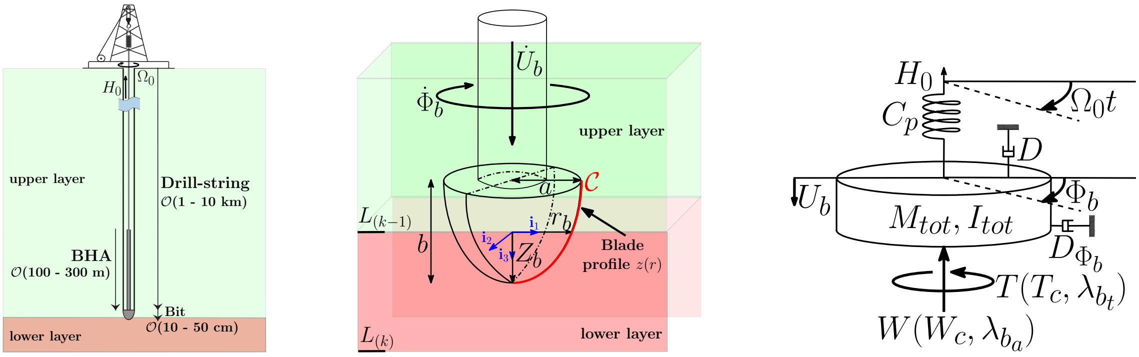

Figure 1: Drilling system schematic for heterogeneous formations (left); the bit transition (middle); model of drill-string system (right).

Dynamic model of drill-string systems in heterogeneous formations

Following the modeling framework in [3] (as depicted in Figure 1-right), the dynamics model of drill-string system incorporating the heterogeneity effect of distinct rock layers is modelled in (scaled) dimensionless form as follows:

$$\mathcal{M}\mathbf{z}'' - \mathcal{H}\left(\tau, \mathbf{z}\ , \mathbf{z}'\ , \mathbf{z}_{\tau_n}\right) = \mathcal{W}\mathcal{L}. \tag{1}$$

Herein, $\mathbf{z}$ is the dimensionless generalized coordinates of the systems and composed by u_b for the axial position of the bit and ϕ_b for the angular position, and all are functions of dimensionless time τ. The matrices in (1) are given by,

$$\mathcal{M} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{z} = \begin{bmatrix} u_b\left(\tau\right) & \phi_b\left(\tau\right) \end{bmatrix}^\top, \quad \mathcal{W} = \begin{bmatrix} \bar{\psi} & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathcal{L} = \begin{bmatrix} \mathcal{L}^{tot}_{k,b_a}\left(\rho_b\right) & \mathcal{L}^{tot}_{k,b_t}\left(\rho_b, \mathcal{L}^{tot}_{k,b_a}\right) \end{bmatrix}^\top, \tag{2}$$

$$\mathcal{H} = \begin{bmatrix} -\gamma {u'}_b \\ \left(-\gamma_\phi\, {\phi'}_b - \left(\phi_b - \omega_0\, \tau\right)\right) \end{bmatrix} + \begin{bmatrix} -\bar{\psi}\ n\ \delta_n\, \bar{g}^\varepsilon_k \left(g^\varepsilon_k \left(1 - \eta^l_\zeta\left(\rho_b\right)\right) + \eta^l_\zeta\left(\rho_b\right)\right) \\ -n\, \delta_n\, \bar{g}^\varepsilon_k\ \left(g^\varepsilon_k\left(1 - {\rho_b}^2\right) + {\rho_b}^2\right) \end{bmatrix} + \begin{bmatrix} \bar{\psi}\, \mathcal{W}_a \\ 0 \end{bmatrix}. \tag{3}$$

The number of layers, with each specific thickness, is K (thus the index of each layer is denoted by $k \in \{2, 3, \ldots, K\}$). The single and double prime symbols denote for the time derivatives of $\mathbf{z}$. $\mathbf{z}_{\tau_n}$ stores the delayed coordinates of $\mathbf{z}$ (i.e., $u_b\left(\tau - \tau_n\right)$, $\phi_b\left(\tau - \tau_n\right)$). τ_n is the (state-dependent) time-delay appearing in the rock cutting process. $\mathcal{M}$ is the mass matrix, and n is the number of bit-blades. Furthermore, one of the pivotal elements in the model is the bit-rock interaction that couples the axial and torsional dynamics of the drill-string and has two main components related to cutting and contact. Matrix $\mathcal{H}$ consists of the generalized forces and torques (except from the components related to the wear-flat contact force and frictional torque), i.e., from the stiffness and damping processes, cutting, gravity and the hook-load imposed by the hoisting/top-drive system at the surface. Thus, we have the torsional damping (γ_ϕ), the axial damping (γ), the angular velocity imposed at the surface rotary table (ω_0), and the resultant of the submerged weight of the drill-string and hook-load ($\mathcal{W}_a$). Moreover, the cutting and contact components are extended for encapsulating the rock heterogeneity effect of two distinct horizontal layers in which the bit may be simultaneously drilling. The lithology parameters influencing these bit force and torque components are the ratio of the intrinsic specific energies between upper and lower layers (g_k^{ε}), the ratio of the frictional coefficients for the contact between upper and lower layers (g_k^{μ}), the ratio of the contact pressures at the bit-rock interface between upper and lower layers (g_k^{σ}), and the ratio between the intrinsic specific energy at the associated layer and its mean values ($\bar{g}_k^{\varepsilon}$). In addition, these bit force and torque components are also affected by the bit parameters related to cutting (η_{ζ}^{l}) and contact (η_{ξ}^{l}), and drill-string design ($\bar{\psi}$). These bit parameters are adapted to the formation layer(s) in which the bit is currently engaged. We use the radius ratio of the bit at the layer boundary (ρ_b as defined in Figure 1-middle) to segregate the portion of the bit engaged in the upper ($(k-1)^{th}$) and lower (k^{th}) layers during the transition. We note that the bit is designed with a particular blade profile ($z(r)$) as a function of its radius r (with maximum radius a). In (3), the depth-of-cut δ_n and the time-delay τ_n are given by,

$$\delta_n = u_b\left(\tau\right) - u_b\left(\tau - \tau_n\right), \qquad \phi_b\left(\tau\right) - \phi_b\left(\tau - \tau_n\right) = \frac{2\pi}{n}. \tag{4}$$

Vector $\mathcal{L}$ stores the dimensionless wear-flat contact force ($\mathcal{L}_{k,b_a}^{tot}$) and the associated frictional torque ($\mathcal{L}_{k,b_t}^{tot}$) acting on the bit wear-flats and is affected by the heterogeneity of the layer(s) in which the bit engages (this dependence is indicated by index k). The generalized directions of these forces and torques are contained in matrix $\mathcal{W}$. These forces and torques obey set-valued force laws (reflecting a unilateral contact law in (5) and a Coulomb friction law in (6)) using a proximal-point formulation on velocity level:

$$\mathcal{L}_{k,b_a}^{tot}\left(\rho_b\right) = \mathrm{prox}_{\bar{C}_a}\left(\mathcal{L}_{k,b_a}^{tot} - r_1\, u'_b\right), \quad \bar{C}_a = \left\{\mathcal{L}_{k,b_a}^{tot}\left(\rho_b\right) \mid -n\, g_{k,\rho_b}^{\sigma}\bar{\mathcal{L}}_{k,b_a} \leqslant \mathcal{L}_{k,b_a}^{tot}\left(\rho_b\right) \leqslant 0\right\}, \tag{5}$$

$$\mathcal{L}_{k,b_t}^{tot}(\rho_b, \mathcal{L}_{k,b_a}^{tot}) = \mathrm{prox}_{\bar{C}_t}\left(\mathcal{L}_{k,b_t}^{tot} - r_2\, \phi'_b\right), \quad \bar{C}_t = \left\{\mathcal{L}_{k,b_t}^{tot}\left(\rho_b\right) \mid -\beta_{k,\rho_b}\mathcal{L}_{k,b_a}^{tot}\left(\rho_b\right) \leqslant \mathcal{L}_{k,b_t}^{tot}\left(\rho_b\right) \leqslant \beta_{k,\rho_b}\mathcal{L}_{k,b_a}^{tot}\left(\rho_b\right)\right\}. \tag{6}$$

$\bar{\mathcal{L}}_{k,b_a}$ is the nominal contact force for the associated rock layer, while g_{k,ρ_b}^{σ} is the value of g_k^{σ} affected by the bit-geometry at the layer boundary (associated to ρ_b). β_{k,ρ_b} is the bit-design parameter that is also affected by the formation heterogeneity and mainly influences the frictional torque. r_1 and r_2 are some positive arbitrary constants.

Preliminary Simulation Results

The simulation results of the dimensionless model of drill-string system for vertical drilling in heterogeneous formations are shown in Figure 2. Under the influence of the heterogeneity, the torque and weight on bit are fluctuating and not reaching the steady-state limit cycles (associated to the responses in homogeneous rock formulations) while the bit moves in heterogeneous and thin layers, i.e., each layer thickness equal to the bit height. The green area is the soft layer, while the red area is for the hard one. These responses show that in the case of heterogeneous formations, the drill-string dynamics can no longer be described through homogeneous models and the developed heterogeneous model is indeed required.

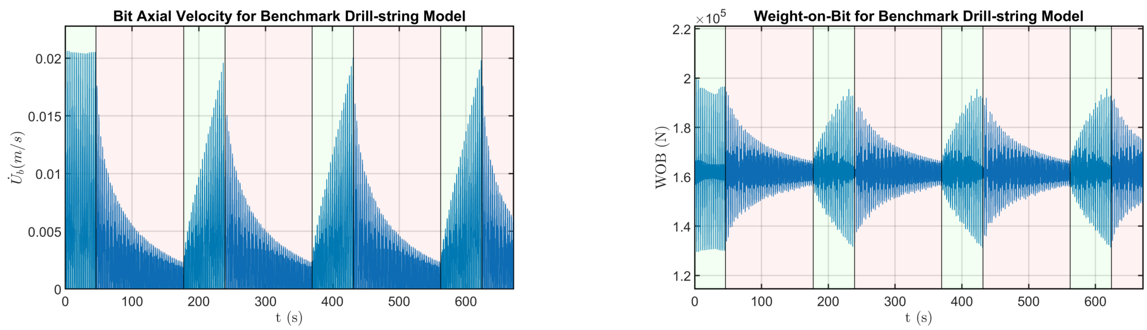

Figure 2: Bit axial velocity/rate-of-penetration in drilling heterogeneous formations (left); weight-on-bit (right).

Conclusion

This abstract presents a drill-string dynamics model with a PDC bit for drilling vertically in the heterogeneous formations with an emphasis on the characterization of the bit-rock interaction during the transition of the bit between two distinct horizontal layers. Next we aim to analyze the drilling efficiency in terms of ROP and to examine the total torsional power losses that can lead to the onset of (torsional) stick slip vibrations in heterogeneous formations.

References

[1] Mensa-Wilmot G., Fear M.J. (2001) The effects of formation hardness, abrasiveness, heterogeneity and hole size on PDC bit performance. *2001 SPE/IADC Drilling Conference, 27 February - 1 March 2001, Amsterdam, Netherlands.*

[2] Detournay E., Richard T., Shepherd M. (2008) Drilling response of drag bits: Theory and experiment. *International Journal of Rock Mechanics and Mining Sciences, Volume 45, Issue 8, pp 1347-1360.*

[3] Wildemans R., Aribowo A.G., Detournay E., van de Wouw N. (2019) Modelling and dynamic analysis of an anti-stall tool in a drilling system including spatial friction. *Nonlinear Dynamics - An International Journal of Nonlinear Dynamics and Chaos in Engineering Systems, Volume 98, Issue 4, pp 2631–2650.*