Analysis of discrete breathers in the mass-in-mass chain in the state of acoustic vacuum

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Summary Present study concerns the dynamics of special localized solutions emerging in the mass-in-mass anharmonic oscillatory chain in the state of acoustic vacuum. Each outer element of the chain incorporates an additional, purely nonlinear mass attachment. Analytical study of the later, revealed the distinct types of stationary discrete breather solutions. Along with the analytical description of their spatial wave profiles we also establish their zones of existence in the space of system parameters. Stability properties of these solutions are assessed through the linear analysis (Floquet). All analytical models are supported by the numerical simulations of the full model.

Introduction

Emergence of spatially localized, time-periodic solutions in the conservative nonlinear system, are known since the pioneering work by Ovchinnikov [1] at 1968. Special localized solutions which are usually referred to as discrete breathers (DBs) remain a subject of broad research interest in the various aspects of modern physics and mechanics. In fact DBs have a well-developed analytical methods when applied to the classical, nonlinear discrete models such as Discrete Klein-Gordon chains (DKGs), Fermi-Pasta Ulam (FPU) models, as well as the Discrete Nonlinear Schrodinger (DNLS) model [2].Formation of spatially localized solutions in all these classical nonlinear models has quite a broad range of applications, including Josephson junctions, nano-mechanical systems, Bose-Einstein condensates, carbon nanotubes, (see for example [3]). Of late, formation of localized excitations as well as nonlinear normal modes in highly nonlinear discrete models admitting a state of acoustic vacuum e.g. purely cubic FPU chains [4], uncompressed granular crystals [5-6], has become a subject of intense research.

Some recent studies, have considered both analytically and numerically the formation of DBs in the two different configurations of locally resonant granular crystals i.e. weakly nonlinear, compressed granular chain [31] as well as the uncompressed ones [32]. Both configurations comprised the chain of granular elements incorporating the internal, linear oscillating inclusions. These numerical and analytical studies unveiled the stationary and mobile DBs and presented a detailed analysis of their stability properties as well as the corresponding bifurcation structures. The system under consideration in the present study qualitatively differs from the previously considered ones by its internal nonlinear, local substructure as well as the special dynamical state of acoustic vacuum. In this study, we focus on the analytic description of stationary discrete breather solutions as well as the prediction of zones of their existence in the space of system parameters.

Model

System under consideration is an infinite, locally resonant chain of elements inter-coupled by linear springs. The governing non-dimensional equations of motion read:

$$\begin{aligned} &\varphi_n'' = (\varphi_{n-1}-\varphi_n)^3-(\varphi_n-\varphi_{n+1})^3-\alpha(\varphi_n-\psi_n)^3 \\ &\varepsilon\psi_n'' = \alpha(\varphi_n-\psi_n)^3 \end{aligned} \tag{1.1}$$

Analysis

Given the homogeneous structure of the system under consideration it is quite natural to study the dynamics and the bifurcation structure of the corresponding standing wave solutions by exploiting the well-known method of separation of variables $\varphi_n(\tau)=\hat{u}_n\eta(\tau),\ \psi_n(\tau)=\hat{v}_n\eta(\tau)$. Where $u=\{\hat{u}_n\}_{n\in Z}, v=\{\hat{v}_n\}_{n\in Z}$ are real sequences and $\eta(\tau)$ is a time-dependent modal coordinate. Introducing this change of coordinates in (1.1) and applying some trivial algebraic manipulations, we obtain the following system of algebraic equations

$$\begin{aligned} &(u_{n-1}+u_n)^3+(u_n+u_{n+1})^3 = u_n-\varepsilon v_n \\ &\alpha(u_n+v_n)^3 = \varepsilon v_n \end{aligned} \tag{1.2}$$

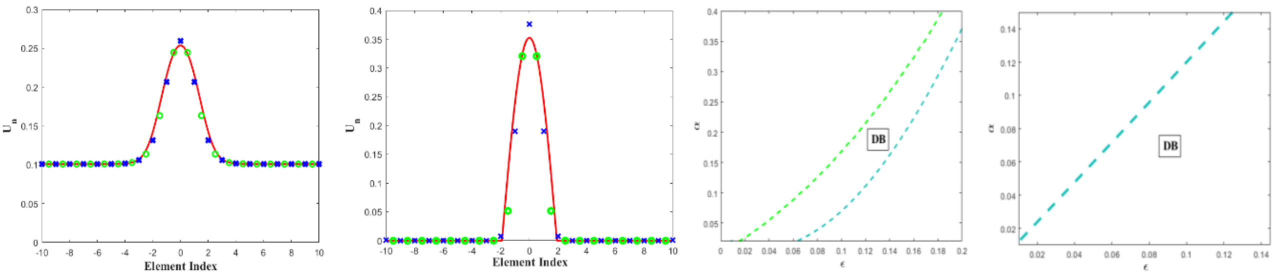
In the present study we construct the analytical description of the spatial profiles of DBs and derive the parametric zones of their existence. These solutions assume the out-of-phase oscillations between the adjacent outer as well as the outer and inner elements. Apparently, analysis of DBs may become extremely cumbersome, if one tries to tackle the system (1.2) as a whole. However, system (1.2) can be considerably simplified if one manages to reduce it from the system involving the amplitudes of vibrations of outer and inner masses $\{u_n\}_{n\in\mathbf{Z}},\{v_n\}_{n\in\mathbf{Z}}$ into the one containing only the motion of the outer elements $\{u_n\}_{n\in\mathbf{Z}}$). Fortunately, any solution of system (1.2) can be effectively represented by the following reduced system

$$
\begin{aligned}
&(u_{n-1}+u_n)^3+(u_n+u_{n+1})^3=u_n-\varepsilon\left(t^{[m_n]}(u_n)\right)^3 \\
&t^{[m]}(u_i)=2\sqrt{\frac{\beta}{3}}\cos\left(\frac{1}{3}\arccos\left(-\frac{3u_i}{2\beta}\sqrt{\frac{3}{\beta}}\right)-\frac{2\pi m}{3}\right),\ m=0,1,2
\end{aligned}
\tag{1.3}
$$

where $m=0,1,2$ corresponds to a certain branch assigned to each one of the cells. In the present study we present the asymptotic description of DBs and establish analytically their zones of existence. Passing to the quasi-continuum limit we obtain the following essentially nonlinear ODEs for each one of the branches,

$$
u\left[u^2\right]_{xx}+\frac{8}{3}u^3+\frac{1}{6}\left\{\varepsilon\left(t^{[m]}(u)\right)^3-u\right\}=0,\ u\le u_{\max}
\tag{1.4}
$$

It can be easily shown that the out-of-phase oscillations, can be obtained on the two branches only, namely ($m=0,1$). Comparison of exact spatial wave profiles of DBs computed from (1.3) with these obtained from QCA (1.4) are presented in Fig. 1 (a) and (b) panels accordingly. In Fig. 1 (c) and (d) we illustrate their zones of existence obtained analytically.



(a) (b) (c) (d)

Figure 1 (a, b) Spatial wave profiles of DBs corresponding to the homogeneous configurations. QCA is denoted with the bold solid line while exact solutions are denoted with ‘cross’ markers for site-centered breathers and ‘o’ markers for the bond-centered breathers. (**a**) $m=0$ (**b**) $m=1$. System parameters: $\varepsilon=0.1,\alpha=0.1$. (c, d) Zones of existence of a discrete breather (DB) corresponding to the homogeneous configurations i.e. m=0,1. **(c)** ($m=0$) **(d)** ($m=1$).

Conclusions

In the present study we analyze the special family of discrete breather solutions. Results of analytical study enable to describe the spatial wave profiles and establish their zones of existence. Separate linear stability analysis of DBs performed in this study revealed their stability zones in the plane of system parameters.

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