Forced vibration analysis of non-linear Euler-Bernoulli beam using Efficient Path Following Method

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Summary. This study focuses on the nonlinear response of dynamical systems due to harmonic force excitation using numerical continuation methods. Nonlinear Normal Modes (NNMs) is a powerful tool for studying behavior of nonlinear vibrational systems. Pseudo arc length continuation method is a very powerful method which is capable to handle strongly nonlinear systems by calculate the NNMs. Several methods are available to compute NNMs. Recently reviewed articles mention the computational cost of the pseudo arc length continuation method that limits its application. Based on assumption presented in other references and eliminate a mathematical construction, in this research used an updating formula to reduce the computational cost of pseudo arc length continuation algorithm. This modified method is called Efficient Path-Following Method (EPFM). In order, forced response of a single degree of freedom duffing system is computed using EPFM method. It seen that this method has decreased computational time considerably up to 70%. The results are in very good conformance to those obtained in other references which shows the accuracy the method. To study the ability of EPFM to handle continues systems, a nonlinear Euler-Bernoulli beam is considered and stable and unstable branches of the solution are computed. It observed that as the nonlinearity of the system gets stronger the updating formula becomes more effective.

Introduction

The structural engineers have utilized modern engineering knowledge to modelling the complex dynamic behavior of structure. For example, rotorcraft, turbomachines, airplanes, bridges, marine platforms, and robotic complex arm have become lighter, this way provide the possibility of operating at higher rates and velocities. Obviously, these On the other hand it could be led to non-linear response of structural systems. The structural response is affected by the nonlinearity. It may also cause complex resonance phenomena. Which makes it impossible to predict system behavior. *Rauscher* and *Mikhlin* develop a definition for modal analysis in nonlinear systems and naming it as "Nonlinear Normal Mode (NNM)". Shaw and Pierre modified the concept of NNM for the class of weakly nonlinear systems. Today, One of the well-known tool to provide a solution of nonlinear dynamical systems is NNMs.[1, 2] Namely, The movement of a system during internal resonance described using NNMs. also nonlinear vibration absorbers designing [2], also providing reduce order model of a nonlinear dynamic system to show system frequency changes and system deformation during free or forced vibration in nonlinear structures [3]. Also it applications include micromechanical oscillators and energy harvesting. Along with analytical methods such as harmonics, multiple scales and other methods for calculating the periodic responses of nonlinear dynamical systems, there are many numerical methods that are preferred because of their less complexity and less mathematical operators. There are many numerical algorithms for computing periodic solution families in nonlinear systems [4, 5].

One of the powerful methods in computation of periodic solution are neumerical continuation methods. *Kerschen* by combination of Shooting method and Pseudo Arc length with continuation presented a High-performance algorithm to calculate the nonlinear normal modes which are defined as periodic solutions of the nonlinear system [3, 6]. Michael W. Sracic in 2010 try to extend the Peeters algorithm to a system with cubic nonlinearity under the harmonic forced excitation [7]. Renson in 2016 has published an reviewed article [8] and studied the various methods for computation of nonlinear normal modes [numerical computation of NNM]. They mention that the "continuation method is a very powerful method for computation of periodic solutions however the computational cost has limited its application to large scale systems".

This paper presents the numerical continuation method modification to speed up the periodic solutions computations. The assumptions and basis of the modified algorithm based on [6, 7], changes made include integrating an updating formula in the algorithm and elimination of phase equation to decrease the computational time of the algorithm. To speed up the process of calculating the periodic solutions we eliminate the phase equation that presented by peeters, and using updating formula presented in [9]. The presented algorithm is called Efficient Path Following Method (EPFM)(Figure 1.f). Numerical experiments show that EPFM is approximately 60-70 percent faster. The theoretical foundations and the algorithm are presented first. Then, a one D.o.F.system and a nonlinear Euler-Bernoulli beam would be studied.

Theory and Algorithm

Governing equation of motion of a N-D.o.F system under harmonic force takes the following form in state space:

$$\dot{\mathbf{z}} = \mathbf{g}\left(\mathbf{z}(\mathbf{z}_0, t, T), t, T\right) = \begin{pmatrix} \dot{\mathbf{x}} \\ -\mathbf{M}^{-1}\left(\mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} + \mathbf{f}_{nl} - \mathbf{F}_e\right) \end{pmatrix} \tag{1}$$

Where $\mathbf{M}$, $\mathbf{K}$, and $\mathbf{C}$ are the mass matrix, stiffness matrix and the damping coefficient matrix, respectively, $\mathbf{f}_{nl}$ is the vector

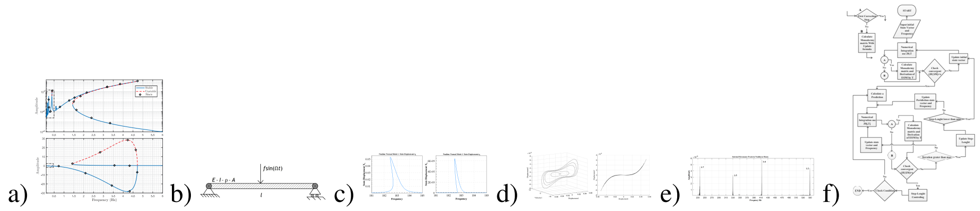

Figure 1: Results figures

of stored nonlinear force and $\mathbf{F}_e$ is the external harmonic force. $\mathbf{z} = [\mathbf{x}\,\dot{\mathbf{x}}]^T_{2N\times 1}$, t is independent variable of time and T is the period of oscillation s. The response of the system to the initial conditions of $\dot{\mathbf{z}}(t=0) = \mathbf{z}_0 = [\mathbf{x}_0\dot{\mathbf{x}}_0]^T$ is represented as $\mathbf{z}(t) = \mathbf{z}(\mathbf{z}_0, t, T)$ to indicate the dependence of the system's response to the initial conditions,$\mathbf{z}(\mathbf{z}_0, t = 0) = \mathbf{z}_0$.
To compute periodic solution integrated Eq.(1) over time interval $[0, T]$. We need to look for the following conditions.

$$\mathbf{H}(\mathbf{z}_0, t, T) \equiv \mathbf{z}_T(\mathbf{z}_0, t, T) - \mathbf{z}_0 \approx 0 \tag{2}$$

Using Newton-Raphson algorithm process try to find periodic condition by expansion of the Eq.(2) as follows:

$$\left.\frac{\partial \mathbf{H}}{\partial \mathbf{z}_0}\right|_{(T,\mathbf{z}_0)} \Delta\mathbf{z}_0 + \left.\frac{\partial \mathbf{H}}{\partial T}\right|_{(T,\mathbf{z}_0)} \Delta T = -H(\mathbf{z}_0, t = T, T) \tag{3}$$

To calculate the periodic response, the equation (3) must be established. The partial derivatives of the equation (3) is calculated based on the procedure given in [7].
The only difference is the calculation of the first derivative which is calculated in the first iteration. Monodromy matrix ($\frac{\partial \mathbf{H}}{\partial \mathbf{z}_0}$) calculated by sensitivity analysis, then in the next iterations, updating formula given in [9] is used to calculate it. the updating formula defined as follows:

$$\left.\frac{\partial \mathbf{H}}{\partial \mathbf{z}_0}\right|^{k+1} = \left.\frac{\partial \mathbf{H}}{\partial \mathbf{z}_0}\right|^{k} + \mathbf{H}^{k+1}\frac{\left(\Delta\mathbf{z}_0^k\right)^T}{|\Delta\mathbf{z}_0|} \tag{4}$$

Results

Considering a system with one degree of freedom governed by Duffing equation presented in [7]. The results show that for force amplitude equal 0.1, EPFM was %52 faster than [7] and for amplitude equal 1.0, EPFM faster about %70, that result show in Figure 1.a. Also a nonlinear Euler-Bernoulli (Figure 1.b) analysed, Figure 1.c depict the periodic solutions compute by EPFM method and Figure 1.d show two dimensional manifold of 1st NNM. EPFM also captured superharmonic and subharmonic peaks as well Figure1.e. in this case EPFM is faster about %60 too.

Conclusions

An effective method is introduced in this study to compute periodic response of nonlinear systems under harmonic excitation. The method is called Efficient Path Following Method (EPFM). The algorithm is a combination of single shooting method and pseudo arc length continuation. The difference between EPFM and other algorithms is that, it uses an updating formula to compute jacobian matrix in correction step. that EPFM also reduces one of the equations in shooting method to speed up computation of periodic responses.
By analyzing the results and computational time, it is concluded that the EPFM is a very effective method for computation of periodic response of nonlinear systems and as the D.o.Fs of the system increase and the nonlinearity of the system gets stronger, this method gets more effective.

References

[1] McConnell KG and Varoto PS (1995) *Vibration testing: theory and practice*. John Wiley & Sons.

[2] Vakakis AF, Gendelman OV, Bergman LA, McFarland DM, Kerschen G and Lee YS (2008) *Nonlinear targeted energy transfer in mechanical and structural systems*, volume 156. Springer Science & Business Media.

[3] Kerschen G, Peeters M, Golinval JC and Vakakis AF (2009) Nonlinear normal modes, part i: A useful framework for the structural dynamicist. *Mechanical Systems and Signal Processing* 23(1): 170–194.

[4] Govaerts WJ (2000) *Numerical methods for bifurcations of dynamical equilibria*, volume 66. Siam.

[5] Seydel R (2009) *Practical bifurcation and stability analysis*, volume 5. Springer Science & Business Media.

[6] Peeters M, Viguié R, Sérandour G, Kerschen G and Golinval JC (2009) Nonlinear normal modes, part ii: Toward a practical computation using numerical continuation techniques. *Mechanical systems and signal processing* 23(1): 195–216.

[7] Sracic MW and Allen MS (2011) Numerical continuation of periodic orbits for harmonically forced nonlinear systems. In: *Civil Engineering Topics, Volume 4*. Springer, pp. 51–69.

[8] Renson L, Kerschen G and Cochelin B (2016) Numerical computation of nonlinear normal modes in mechanical engineering. *Journal of Sound and Vibration* 364: 177–206.

[9] Jelveh M (2015) *Analysis of Nonlinear dynamic systems using nonlinear normal modes*. PhD Thesis, Amirkabir University of Technology.