Computation of Damped Nonlinear Normal Modes Using Force Appropriation Technique and Efficient Path Following Method

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Summary. In this paper a new method is presented for computation of nonlinear normal modes (NNMs) of damped nonlinear systems. Pseudo arc length continuation of periodic solution method is well accepted as a strong tool for computation of NNMs and in many literature is used as reference solution. However, this method is limited to conservative systems and its application to damped nonlinear systems is still an open field of problem and very few attempts were made to solve that. In this article a new algorithm based on combination of force appropriation technique and a modified version of pseudo arc length continuation method called efficient path following method (EPFM) is presented to compute NNMs of damped nonlinear system. In order to investigate the capability of the algorithm, NNMs of a two D.o.Fs damped mass-spring system was calculated. It was observed that the results were in very good agreement to those obtained in other references.

Introduction

In recent decade, NNMs have attracted many researchers and comprehensive article reviews are published on the concept, theory and application of them. Nowadays, there are two common definitions for NNMs: a) periodic motion of nonlinear autonomous system b) two dimensional invariant manifold in phase space. As Renson et al. mentioned, The role played by damping in the dynamics of nonlinear structures is not yet completely uncovered and its effect on the modal interactions and invariant manifolds are largely unanswered [5].

Pseudo arc length continuation is used to compute NNMs based on periodic motion definition of them. However in general autonomous motions of damped systems are not periodic. This makes it difficult to use pseudo arc length continuation for computation of damped NNM. Kuether and Allen used combination of Force appropriation technique, complex Fourier expansion and numerical continuation to compute NNMs of damped systems [3]. They compute NNMs up to moderate levels of energy, but they couldn't capture internal resonances. Krack added an artificial damping proportional to mass matrix to the equation of motion of the system with suitable sign to cancel the effect of system's damping[2]. By adding this artificial damping, he made the solution of the system periodic and used pseudo arc length continuation for computation of NNMs. This method was limited to low damped systems and he didn't capture any modal interactions.

In this paper a novel method is presented based on force appropriation technique and EPFM. EPFM is a modified version of pseudo arc length continuation method which Sadr et al. showed that, it is up to 70% faster than similar algorithms [4]. This new method is capable to compute damped NNMs and their interactions with no limitation on energy level. In order to use this algorithm

Theory and application

One of the most important properties of NNMs is that force resonances occur near them. The governing equation of motion under harmonic force in state space is written as equation (1) in which z is vector of state variables defined as $Z^T = [\dot{x}^T \quad x^T]$ where $\dot{x}$ is velocity vector and x is vector of displacements. F_0 is the amplitude of external force, V_F is the shape of external force and ω is frequency of external force which could be defined as $\omega = \frac{2\pi}{T}$ where T is the period of the external force.

$$\dot{Z} = g(Z) + F_0 V_F \sin(\omega t) \quad (1)$$

The shooting function would be defined as follow:

$$H(Z_0.T.F_0) = Z(T) - Z_0 \quad (2)$$

So the Tylor expansion of H and correction vector of initial guess maybe found by formula of equation (4). The matrix $\frac{\partial H}{\partial Z_0}$ and $\frac{\partial H}{\partial T}$ are computed according to [4] and $\frac{\partial H}{\partial F_0}$ is computed by means of finite difference method.

$$H(Z_0.T.F_0) + \frac{\partial H}{\partial Z_0}\Delta Z_0 + \frac{\partial H}{\partial T}\Delta T + \frac{\partial H}{\partial F_0}\Delta F_0 + HOT = 0 \quad (3)$$

$$\frac{\partial H}{\partial Z_0}\Delta Z_0^k + \frac{\partial H}{\partial T}\Delta T^k + \frac{\partial H}{\partial F_0}\Delta F_0^k = -H^k \quad (4)$$

$$Z_0^{k+1} = Z_0^k + \Delta Z_0^k \,;\; T^{k+1} = T^k + \Delta T^k;\; F_0^{k+1} = F_0^k + \Delta F_0^k \quad (5)$$

Among all possible periodic solution of forced vibration, solutions satisfying phase quadrature lag criterion are NNMs of the nonlinear systems. It means that each harmonic in the excitation must be 90 degrees out of phase with each harmonic in the response. From energy point of view, phase quadrature lag criterion is defined as equation (6)[1].

$$\int_0^T \dot{x}^T C \dot{x}\, dt = \int_0^T \dot{x}^T F(t)\, dt \quad (6)$$

Because the periodic solution of the system is invariant wrt linear shift of time origin, a phase condition equation may be used. In pseudo arc length continuation method, to make the convergence faster, the correction vector is forced to be perpendicular to the prediction vector. So the correction vector is calculated from following set of equations in which j is the continuation index and k is the iteration index in each step of continuation. Prediction vector of each continuation step is defined as $P = \begin{bmatrix} P_{Z_{0\,j}} & P_{T_j} & P_{F_j} \end{bmatrix}$.

$$\begin{bmatrix} \frac{\partial H}{\partial Z_0} & \frac{\partial H}{\partial T} & \frac{\partial H}{\partial F} \\ h(Z_0) & 0 & 0 \\ P_{Z_{0\,j}} & P_{T_j} & P_{F_j} \end{bmatrix} \begin{Bmatrix} \Delta Z_0^k \\ \Delta T^k \\ \Delta F^k \end{Bmatrix} = \begin{Bmatrix} -H \\ 0 \\ 0 \end{Bmatrix} \quad (7)$$

In order to implement the algorithm, forced response of the system at two near constant force amplitude are calculated using method developed by Sadr et al. and solutions satisfying energy condition defined by equation (8) are selected. ε is the acceptable tolerance. Then prediction vector is constructed using these tow point. Then continuation procedure is applied as stated before.

$$E_{err} = \left| \frac{\int_0^T \dot{x}^T C \dot{x}\, dt - \int_0^T \dot{x}^T F(t)\, dt}{Mechanical\ energy} \right| < \varepsilon \quad (8)$$

Results

The method is applied to a two D.o.Fs Damped cubic nonlinear mass spring system and its first NNM is investigated. As stated in many references, NNMs of very lightly damped systems are similar to that of corresponding conservative systems. As seen in the results, when the damping is very low, the NNM is very close to conservative NNM. As the damping increases the modal interaction tong gets shorter and wider. It is observed that, the obtained solutions completely satisfy energy condition of equation4.

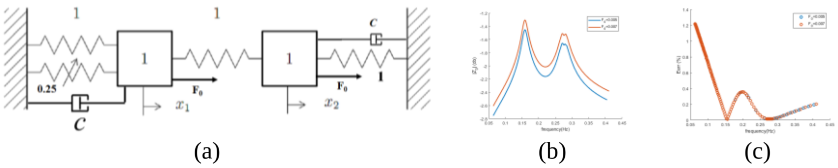

Figure 1: (a) damped cubic nonlinear mass spring system (b) forced response of the system at constant amplitude of force (c)energy condition of the forced response

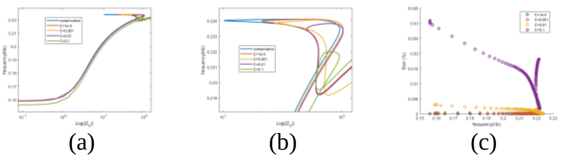

Figure 1: (a) FEP of First NNM for different values of damping (b) Detail view of first NNM (c)energy condition for obtained results

conclusion

in this paper a new method based on force appropriation technique and pseudo arc length continuation is presented for computation of damped NNMs of nonlinear system. The method applied to a two D.o.Fs system. As observed the accuracy of results was very good. The method can capture the modal interactions without any difficulties. It was seen from the results as the damping of the system increases, the FEP tongs becomes shorter and wider.

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