# Non-linear Vibrations in a Coiling Process with Periodically Changing Radius

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Summary. The mechanical model of a winding process has to consider the coupling of the vibrations of the strip and the coiling drum due to non-steady state operation conditions. In the mechanical model of this variable mass system additionally variable parameters are present and result in non-linear equations of motion. The longitudinal and transversal motion of the axially moving strip and the bending deflection of the coiling drum are considered by Rayleigh-Ritz approximations which involve the application of the extended equation of Lagrange. A periodically changing radius is a potential source of vibration excitation. A time integration algorithm with small time steps has to guarantee a converged solution for the long computation time. Simulation results of steady state and non-steady state operation conditions are computed and show the coupling of the vibrations and the excitation due to the periodic radius function.

## Introduction

A suitable mechanical model is necessary for the simulation of the vibrations in a coiling process. In the coiling process an axially moving strip moves continuously towards a rotating drum where it is coiled. Between two successive coiling processes the strip passes through a Steckel mill where the thickness is reduced. The mechanical model starts at the exit of the Steckel mill and considers the axial motion of the strip with the transversal oscillations. Then the strip is coiled where the strip is attached on the drum, contributes to the bending stiffness and increases the mass of the drum. The resulting mechanical model is a non-linear dynamic model with varying mass and system parameters, which are defined by the variable outer radius of the drum, the variable bending stiffness of the drum and a variable eccentricity of the rotating drum. Due to the coiled material the mass of the coiling drum increases or decreases continuously. For the outer radius of the coiling drum an Archimedian Spiral and a periodic step function is assumed, which gives an outer radius and bending stiffness depending on the coiled strip length. For the simulation of the coiling process with the long computation time a integration algorithm was implemented. For the derivation of the equations of motion Rayleigh-Ritz approximations are used to get only a few degrees of freedom in the mechanical model. The application of the extended equations of Lagrange, see [1], is necessary as the mass in the system is not constant, which is a restriction for the well-known equations of Lagrange, see [2]. In the extended equations of Lagrange the control volume concept with the surface integrals with partial derivatives as a kernel are present. In [3] additionally some literature with examples on dynamic systems with variable mass is discussed. In [4] an alternative approach for the influence of the variable mass is considered using reactive forces, where also some examples are shown. The coupled vibrations are analysed and numerical studies are performed in order to increase the knowledge about the complicated variable mass non-linear dynamic system of the coiling drum with an outer radius involving a periodic excitation and the axially moving strip. For the dynamic system the initial and boundary conditions are defined and with the given operation conditions a time-integration algorithm computes the solution.

## Mechanical modelling of the coiling drum and the moving strip

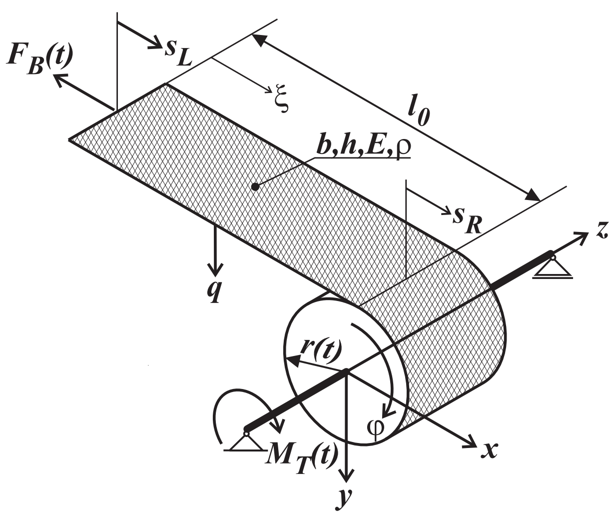


Figure 1: Mechanical model of the rotating drum with the axially moving strip

The mechanical model includes the coiling drum on elastic bearings and the moving strip, see Fig. 1. Rayleigh-Ritz approximations and the extended equations of Lagrange have been used for the derivation of the mechanical model. For the derivation of the equations of motion it is important to distinguish between the material control volume and the spatial control volume. The mechanical model has five degrees of freedom, the horizontal and vertical deflection $x$, $y$ and the rotation angle $\varphi$ of the coiling drum, the transversal deflection of the moving strip $q$ and the entrance speed of the strip $s_L$. The strip tension force $F_B$ is given as a predefined time-dependent value at the entrance of the system and the torque at the coiling drum $M_T$ is computed. The coiling drum is modeled as a beam with different stiffness in longitudinal direction. The outer radius of the drum increases and an Archimedian spiral $r = r_0 + \frac{h\varphi}{2\pi}$ as well as a step-function $r = r_0 + h \operatorname{floor}\left(\frac{\varphi}{2\pi}\right)$ have been analysed. For the computation of the actual stiffness it is assumed that the coiled strip is attached to the drum and contributes to the stiffness.

The model of the coiling drum and the moving strip is described and derived in [5]. The coupling between the coiling drum and the moving strip is modelled considering the strain in the strip $\varepsilon_S = \varepsilon_{xx} - zw'' + \frac{1}{2}w'^2$. The horizontal motion of the strip in longitudinal direction at the right position where it touches the coiling drum considers the shortening effect of second order, see [5]. $\varphi$ ist the rotation angle and $x$ is the horizontal deflection of the center of the rotating drum.

## Computed Results

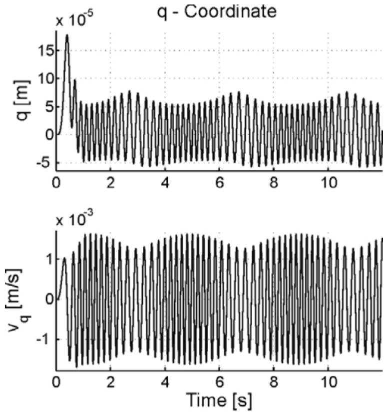


Figure 2: Amplitude and velocity of transversal motion of the strip

For the derived mechanical model the solution was computed and a parametric study has been performed. The parameters of the coiling drum based on the computations presented in this contribution are $L_0 = 5$m, $r_0 = 0.45$m, $h = 10$mm, $b = 0.5$m, $E = 105$kN/mm$^2$, $c_C = 10^7$kN/m, $\rho = 7800$kg/m$^3$, $m_0 = 1200$kg. For a strip tension force of $F_B = F_{B0}\left(1 + \frac{\sin(\pi t/2)}{2}\right)$ with $F_{B0} = 50$kN the computation is carried out. From the results of the amplitude of the transversal strip vibrations in Fig. 2 the non-linear coupling effect with the varying frequency and amplitude is shown.
When a step-function of the outer radius of the coiling drum is used, the computed resulting vibrations are shown in Fig. 3 and Fig. 4 for a constant strip tension force at the entrance of the system in Fig. 1. It can be seen that the effect of the step function in the outer radius gives an impact-like excitation which occurs after every revolution. For successive rotations the vibration amplitudes are computed with a small time step to get a convergent result as the step-function of the radius gives a modification in the kinematics of the system.

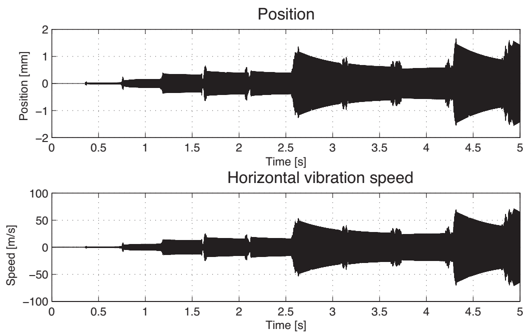


Figure 3: Horizontal Position and speed of the Center of the Coiling Drum

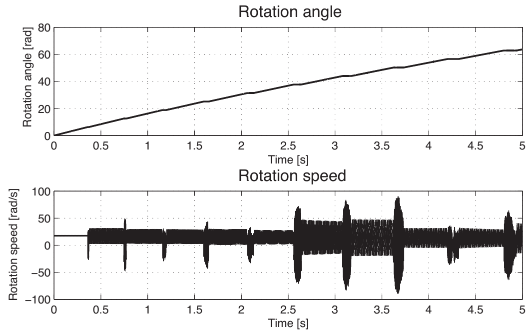


Figure 4: Rotation angle and speed of the coiling drum

## Conclusion

A mechanical model with a variable mass and varying parameters and a periodic function for the outer radius of a coiling process was derived. The simulation results show a production process for a constant axial speed. For a defined variation of the strip tension force at the entrance the vibration amplitudes of the coordinates show non-linear coupled vibrations and the frequency and amplitude for the transversal strip oscillation depend on the strip tension force. For the step-function of the outer radius the computation needs a higher effort and shows an excitation after every revolution of the coiling drum.

## Acknowledgement

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## References

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