Asymmetric oscillations of a shallow spherical cap under a harmonic pressure field: bifurcations and chaos

Giovanni Iarriccio, Antonio Zippo and Francesco Pellicano
Dept. of Engineering "Enzo Ferrari", University of Modena and Reggio Emilia, Modena, Italy
Intermech Mo.Re Centre, Modena, Italy

Summary. The nonlinear vibrations of a homogeneous, isotropic, and shallow spherical cap under a harmonic pressure field are investigated. The problem is tackled using a semi-analytical method based on Novozhilov's nonlinear thin shell theory. The partial differential equations (PDEs) are reduced to a set of ordinary differential equations (ODEs) through the Rayleigh-Ritz approach and Lagrange equations. The final equations of motion are numerically solved using both continuation and direct integration techniques. Results depicted the activation of non-symmetric vibrational states, with the presence of multiple bifurcations and chaotically-modulated oscillations.

Introduction

The characterization of thin-walled structures under dynamic loads has always received considerable research attention due to the large number of applications in Engineering from macro to nanoscales (e.g. propellant tanks, micro-electro-mechanical systems, nanotubes).
This study proposes a method to analyze the dynamics of a homogeneous, isotropic, and thin-walled shallow spherical cap under a uniform, time-varying pressure distribution, see Figure 1.
Despite most of the previous studies on this topic being limited to the axisymmetric vibrations, which neglected the possible onset of non-symmetric vibrations, some authors suggest retaining asymmetric modes into the analysis to improve the matching among experimental and numerical results [1].
To this end, the formulation here presented uses Novozhilov's thin shell theory [2], a follower pressure to model the external load distribution [3], and it retains the asymmetric modes into the reduced-order model. Complete details regarding the present study are reported in Refs [4,5].

(a) (b)

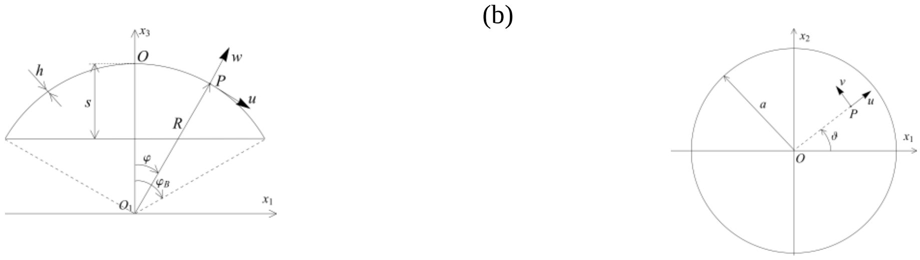

Figure 1: Spherical cap reference system: (a) side and (b) top view.

Results and Discussion

Following the analyses presented in Refs [4,5], a 38 dofs reduced-order model is considered. The structure is loaded by a uniform static pressure superimposed to a harmonic one. The frequency of the harmonic pressure varies about the first axisymmetric mode resonant frequency $\omega_{1,0}$. Using a continuation method to analyze the stability of periodic solutions, and directly integrating the equations of motion, a dynamic scenario characterized by multiple bifurcations and chaos is shown.
In Figure 2, the frequency-response diagram of the asymmetric modal coordinate $f_{w,1,2}^{(d)}$ is given. The continuation method (black-solid line) shows the presence of period-doubling bifurcations (PD) leading to the onset of asymmetric oscillations. The stability loss of the periodic responses (black-dotted line) agrees with the irregular response trend prompted by the direct integration of the equations of motion (red-dashed/blue-solid line).
Since an irregular frequency-response diagram suggests chaotic oscillations, Figure 3 shows the time response of $f_{w,1,2}^{(d)}$ for a normalized forcing frequency $\Omega = 1.09 \cdot \omega_{1,0}$. Two different phenomena could be observed: a slow dynamic, which module the oscillation amplitude, and a fast dynamic governed by rapid burst. This pattern has some periodicity, and the evolution of the Poincaré map clearly shows weekly chaotic vibrations: by reporting the section for an increasing number of periods n_P, the sparse cloud contour becomes regular, and a dense kernel becomes noticeable.

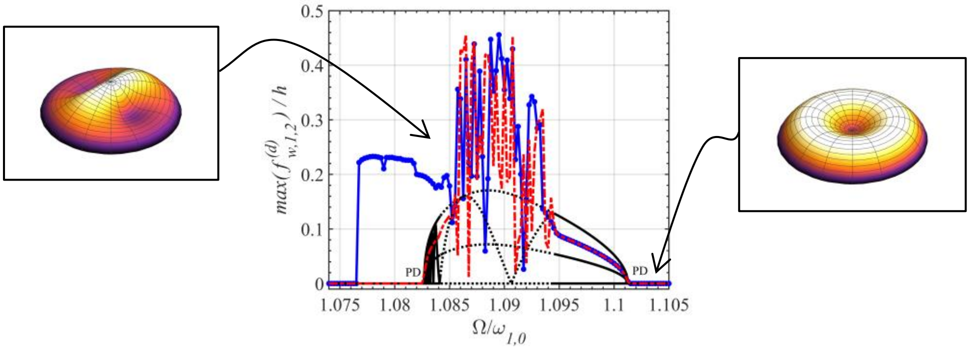

Figure 2: Asymmetric mode activation after period doubling bifurcation: continuation method VS direct time integration.

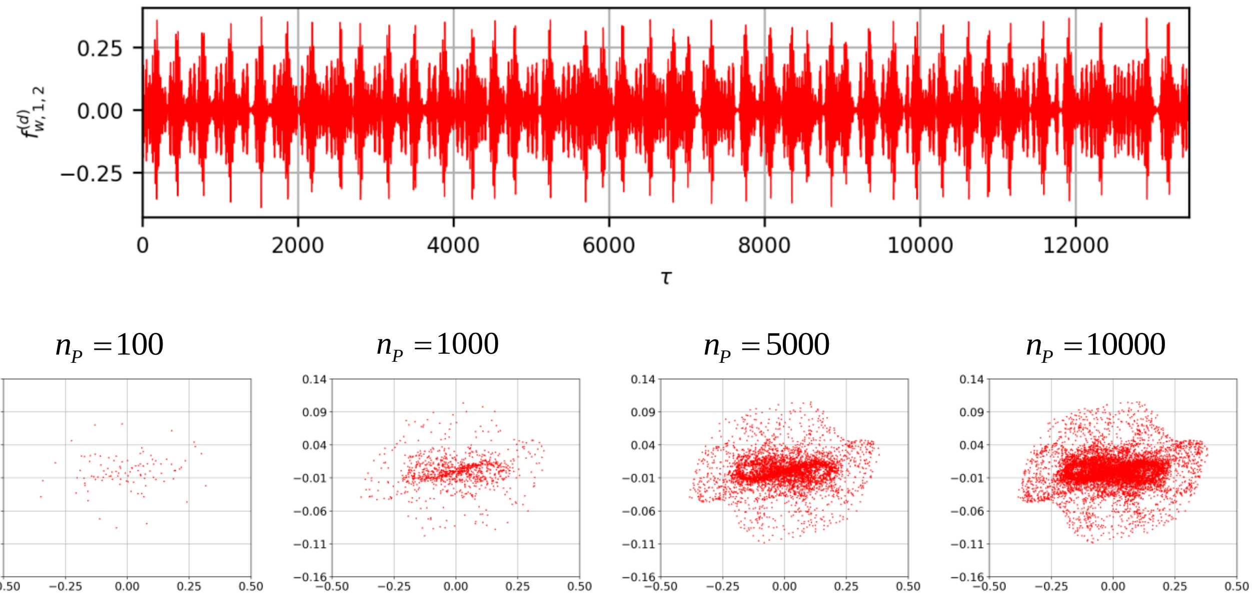

Figure 3: Chaotically-modulated oscillations with fast and weekly periodic burst: time history and progressive Poicaré map evolution of the asymmetric modal coordinate and its time derivative.

References

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