Dynamic stability of tuned vibration absorbers allowing a rotational mobility

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Summary. Rotating machines are often subject to fluctuating torques, leading to vibrations of the rotor and finally to premature fatigue and noise pollution. These vibrations can be reduced using tuned vibration absorbers (TVA). These passive devices are made of several masses oscillating along a given trajectory relative to the rotor. Previous studies showed that the dynamics of these devices is subject to instabilities. In this paper, the dynamic stability of a new class of TVA is investigated. The particularity of this new class is that the TVA now admits a significant rotation motion relative to the rotor, in addition to the traditional translation motion. Efficiency of such devices is optimal for a perfect synchronous motion of oscillating masses. However, due to non linearities, masses unisson can be broken for the benefit of energy localization on a given absorber, leading to a loss of mitigation performances. To assess the stability of such devices, a dynamical model based on an analytic perturbation method is established. The aim of this model is to predict analytically localisation and jumps of the response. The validity of the model is confirmed through a comparison with a numerical resolution of the system's dynamics.

Presentation of the system

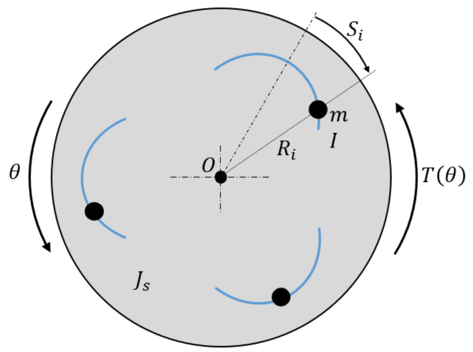

Figure 1: Scheme of the system studied for $N = 3$ absorbers

The system considered in this study consist of a rotor (or support plate) of inertia J_s rotating about its center O with a mean velocity Ω with respect to the galilean frame. Its angular position is written θ. On this rotor are articulated N identical absorbers of mass m and inertia I. These absorbers are located along their trajectory through the curvilinear abscissa S_i. The distance from point O to the mass center of the i^{th} absorber is written $R_i(S_i)$, $i = 1, \dots N$. The shape of the trajectory is controled through this function. In addition to the translation motion along the trajectory, the rotation of the absorbers about their mass center with respect to the rotor frame is considered in this study, and denoted by $\alpha_i(S_i)$. A mechanical device enables to prescribe it as a function of S_i. We choose R_i (respectively α_i) to be a symmetric (respectively antisymmetric) function as it is the case in practice due to design issues. Finally, this system is subject to an external tork $T(\theta)$. In this framework, the equations of the system take the form:

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{f_{in}}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{C}\dot{\boldsymbol{q}} + \boldsymbol{f_{int}}(\boldsymbol{q}) = \boldsymbol{F}\cos(n_e\theta)\,, \tag{1}$$

where $\boldsymbol{q} = [\theta, ..., S_i, ...]^T$ is the vector of unknowns (T denotes the transpose). $\boldsymbol{M}(\boldsymbol{q})$ is the mass matrix and depends on $\boldsymbol{q}$. $\boldsymbol{C}$ is the damping matrix. $\boldsymbol{f_{in}}(\boldsymbol{q}, \dot{\boldsymbol{q}})$ is the inertial forces vector, including Coriolis terms, and depends on $\boldsymbol{q}$ and $\dot{\boldsymbol{q}}$. $\boldsymbol{f_{int}}(\boldsymbol{q})$ is the internal forces vector and depends on $\boldsymbol{q}$. The external forces vector is $\boldsymbol{F} = [T_1, ..., 0, ...]^T$ where T_1 is the amplitude of the fundamental harmonic of the external torque. Finally, n_e is the excitation order and it can be regarded as a proportionality coefficient between the excitation frequency and the mean rotation speed.
Performing a linear study of this system, one can find that the $N+1$ eigenfrequencies can be written $\omega_0 < \omega_1 < \omega_2$ where ω_1 is of multiplicity $N-1$ and is also the natural frequency of the absorbers uncoupled from the rotor. The corresponding eigenmodes read $\phi_0 = [1, 0, \dots, 0]^T$, $\phi_2 = [-a, 1, \dots, 1]^T, a > 0$. ϕ_0 is a rigid body mode for which the absorbers are not excited and ϕ_2 is a mode for which absorbers are in-phase and in phase opposition with respect to the rotor. The mode shapes corresponding to the $N-1$ degenerated modes of eigenfrequency ω_1 all have a zero first component so that this mode is not excited by the external torque. For instance, for $N = 2$, $\phi_1 = [0, 1, -1]^T$: the absorbers are out of phase.

Non-linear study

In practice, the absorbers create an antiresonance of the rotor at frequency ω_1, which is thus the chosen operating point of the system [1]. In the following, a non-linear study of the steady-state response of the system is carried out in the vicinity of this operating point and we are interested in the response of the absorbers. An analytical model representing the dynamical behaviour of the system is developped. Following [2], the first step to build the model is to perform a change of the independent variable from t to θ and a scaling of the parameters. Then, to continue the study, we choose in this work to use the multiple scale method [3]. This approach leads to the obtention of a system of the form

$$\begin{cases} D_1 A_i = f_1(A_i, A_j, \varphi_i, \varphi_j) \\ D_1 \varphi_i = f_2(A_i, A_j, \varphi_i, \varphi_j) \end{cases} \tag{2}$$

where D_1 represents the derivative with respect to the slow time, and A_i and φ_i are the amplitude and phase of the first harmonic of the scaled i^{th} absorber's response. On the one hand, using the fact that the stable response of the absorbers is governed by the in-phase mode, one can solve (2) to get the unisson response of the absorbers. On the other hand, the stability of (2) can be assessed through the computation of the determinant of its jacobian [4]. This leads to the obtention of two bifurcation curves. The intersection of these curves with the absorbers' response indicates a stability change towards either a non-synchronous response of the absorbers (i.e. the absorbers stop moving in unisson) or an unstable periodic response (this leads to jumps phenomena).

Results and introduction of the design space

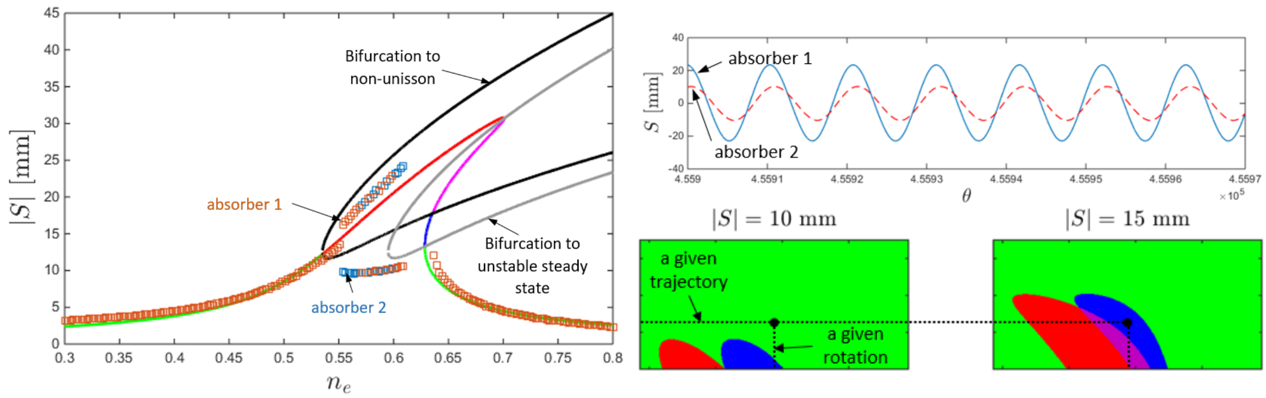

Figure 2: Left hand side: Analytic and numerical comparison of the absorbers' order response. Squares are numerical results. Top right hand side: Time signal of a non-unisson motion of the absorbers. Bottom right hand side: maps of designs representing their associated stability. The black dot corresponds to a given design (i.e. a given trajectory and a given rotation function). A color code indicates the stability of the response. Green: stable; red: unisson unstable; blue: steady state unstable; purple: unisson and steady state unstable

We choose here to apply the above study on a system of $N = 2$ absorbers. Fig.2 presents the response of the absorbers as a function of the excitation order, which has the meaning of a driving frequency. In this case, ω_1 corresponds to $n_e = 0.5$ and the resonance to ω_2. The numerical results are obtained through time integrations of the equations of motion using a Runge-Kutta algorithm. Like what was observed in the case of absorbers with a pure translation motion [5], results show that the system looses its stability through a pitchfork bifurcation in favor of a non-synchronous response. The model allows to accurately predict the bifurcation point, which is theoretically located at the intersction of the black curve (limit of unisson unstability region) and the unisson response. The time signal shown in Fig.2 clearly shows the difference in amplitude and in phase of the response of the non-synchronous response of the absorbers.
For a given excitation order and absorbers' amplitude, one can use the analytical model to assess the stability of several designs of TVA simultaneously. Designs can be defined by their associated trajectory and rotation functions, so that the result takes the form of 2D maps as it is shown on Fig.2. The black dot represented on these maps is fixed, as it is a given design. We can see that depending on the absorbers' amplitude of motion, this design goes through different stability states. These maps can be very efficient to determine whether a given design is prone to desynchronisation and/or jumps.

Conclusions

This study presents analytical and numerical results about non-linear localisation and jumps in a TVA. This work generalises previous studies to TVA with absorbers admitting a rotation about their mass center. The analytical and numerical results agree very well, showing that the model developped is able to give a good representation of the physical system. The design space has been introduced, allowing a quick assessment of the efficiency of TVA designs.

References

[1] A. Renault, O. Thomas, H. Mahé (2018) Numerical antiresonance continuation of structural systems. *Mechanical Systems and Signal Processing* **116**:963-984.

[2] A. Alsuwaiyan and S. Shaw (2002) Performance and dynamic stability of general-path centrifugal pendulum vibration absorbers. *Journal of Sound and Vibration* **252**:791-815.

[3] A. Nayfeh (2000) Perturbation methods. *WILEY*.

[4] J. Issa and S. Shaw (2015) Synchronous and non-synchronous responses of systems with multiple identical nonlinear vibration absorbers. *Journal of Sound and Vibration* **348**:105-125.

[5] A. Grolet, A. Renault and O. Thomas (2017) Energy localisation in periodic structures: application to centrifugal pendulum vibration absorbers. *ISROMAC 2017, Maui, Hawaii.*