Study of the behaviour of the trombone using bifurcation diagrams

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Abstract

In this study, an acoustic resonator – a bass trombone – with multiple resonances coupled to an exciter – the player's lips – with one resonance is modelled by a multidimensional dynamical system, and studied using the continuation and bifurcation software AUTO [1]. Bifurcation diagrams are explored with respect to the blowing pressure, with focus on the minimal blowing pressure giving stable periodic regime.

Brass instruments can be described thanks to both linear and nonlinear mechanisms: a localised nonlinear element (the lips' valve effect) excites a passive linear acoustic multimode element (the musical instrument, usually characterised by its input impedance in the frequency domain); the latter acoustic resonator in turn exerts a retroaction on the former mechanical resonator. Such musical instruments are self-sustained oscillators: they generate an oscillating acoustic pressure (the note played) from a static overpressure in the player's mouth (the blowing pressure).

A brass instrument having N acoustic modes and M mechanical modes can be mathematically modelled as an autonomous nonlinear dynamical system of dimension $2(N+M)$ [2, 3]. On the one hand, the corresponding acoustic input impedance of the resonator is defined in the Fourier domain by the ratio between the pressure and the volume flow at the input of the instrument :

$$P(\omega) = Z(\omega)U(\omega) \tag{1}$$

with ω the angular frequency. The modulus of Z is shown figure 1 for a bass trombone. Since AUTO requires equations to be expressed in the time domain, a modal decomposition is performed on the measured input impedance of a bass trombone in order to obtain an analytical form for $Z(\omega)$, thus allowing equation (1) to be expressed in the time domain by taking its inverse Fourier transform.

On the other hand, the nonlinear behaviour of the lips can be modelled by the relation between the volume flow $u(t)$ and the acoustic pressure $p(t)$:

$$u(t) = wh(t)\,\Theta(h)\,\mathrm{sgn}\left(p_m - p(t)\right)\sqrt{\frac{2\,|p_m - p(t)|}{\rho}} \tag{2}$$

with w and $h(t)$ the width and the height of the lips' aperture respectively, p_m the mouth pressure, and ρ the air density. The Heaviside distribution Θ accounts for the fact that lips cannot interpenetrate: as soon as the lips touch ($h = 0$), the volume flow is forced to zero. The sign function sgn accounts for the possibility of having an airflow going from the instrument into the player's mouth, in the absence of experimental evidence to the contrary.

Eventually, $h(t)$ can be linked to $p(t)$ to close the system by the following equation, which accounts for the resonance of the lips :

$$\ddot{h} + \frac{\omega_l}{Q_l}\dot{h} + \omega_l^2\,(h - h_0) = \frac{p_m - p}{\mu} \tag{3}$$

with ω_l and Q_l being respectively the frequency resonance and the quality factor of the lips, and μ the lips' mass per unit area.

In this study, the input impedance of the brass instrument addressed – a bass trombone – will be described by $N = 12$ acoustic modes, and the exciter – the player's lips – will be assumed to have only one mechanical mode ($M = 1$), resulting in a 26-dimensional dynamical system. It is worth noting that this minimal brass model does not account for the « brassy » sound produced by a trombone, which requires a nonlinear description of sound propagation in the instrument to be addressed.

The behaviour of the instrument can be first studied close to its static equilibrium using the Linear Stability Analysis (LSA), so as to determine if oscillatory regimes (corresponding to the instrument producing a sound) could potentially arise from destabilised equilibrium eigenvalues of the linearised system. Such an approach has already been carried out in [4], especially to characterise the ease of playing of a brass instrument. Indeed, the latter can be related as a first approximation to the linear threshold pressure (figure 2a), since the lower the threshold pressure, the lower the physical effort the player has to make to play a note.

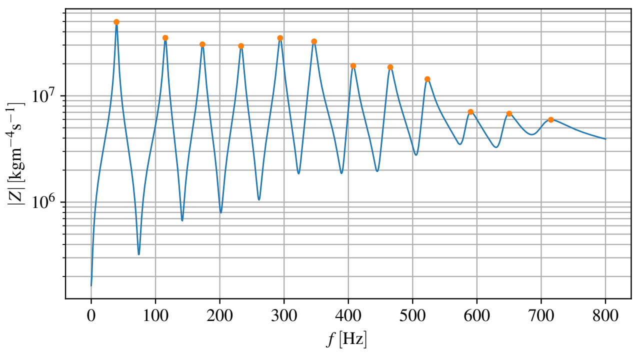
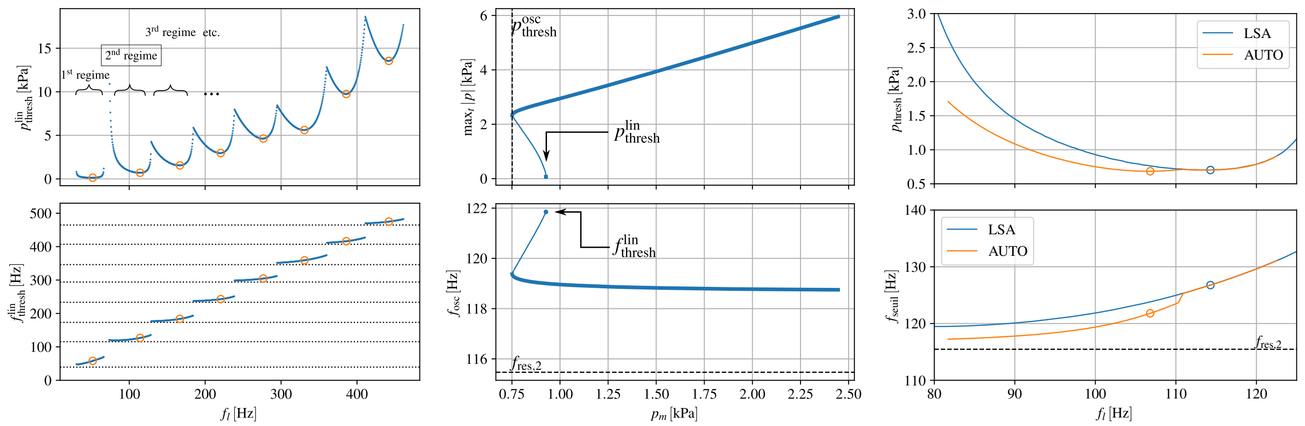


Figure 1: Modulus of the input impedance of a bass trombone vs. frequency. 12 resonances (orange dots) can be seen, corresponding to $N = 12$ acoustic modes.

The ease of playing is then assessed in more depth through the study of the dynamic behaviour of the instrument, using the continuation and bifurcation software AUTO [1] as in [5] or using an alternative continuation method (Manlab) as in [6], thanks to which bifurcation diagrams are explored with respect to the blowing pressure for instance. The oscillation threshold pressure can then be infered from such a diagram (figure 2b). It is worth noticing that the oscillation threshold pressure is not necessarily equal to the linear threshold pressure, as it is the case on figure 2b. It then results in an actual threshold pressure lower than the linear threshold pressure given by the LSA (figure 2c), hence the need to take into account the dynamic behaviour of the system. Also, the n^{th} regimes are all direct Hopf bifurcations for $n > 5$, thus implying $p^{\text{osc}}_{\text{thresh}} = p^{\text{lin}}_{\text{thresh}}$ for these.



(a) LSA of a bass trombone

(b) Bifurcation diagram (2$^{\text{nd}}$ regime of a bass trombone; $f_l = 100$ Hz)

(c) Superposition of p_{thresh} given by the LSA and AUTO (2$^{\text{nd}}$ regime of a bass trombone; $f_l \in [80\,\text{Hz}, 125\,\text{Hz}]$)

Figure 2: Linear Stability Analysis (left), bifurcation diagrams (middle), and comparison of threshold pressure values given by the LSA and AUTO (right), corresponding to the case of the input impedance of a bass trombone shown figure 1. Left: top and bottom plots represent respectively the linear threshold pressure and frequency vs. the lip frequency. The orange circles identify the value of f_l corresponding to a local minimum of the curve (easiest note to obtain), and dotted lines represent the resonance frequencies of the input impedance for comparison. Middle: Top and bottom plots represent respectively the maximum amplitude of the periodic oscillation branches and the frequency of the corresponding periodic solutions vs. the blowing pressure. Right: top and bottom plots represent respectively the threshold pressures and frequencies given by the LSA and AUTO vs. the lip frequency. As for figure 2a, the circles identify the value of f_l corresponding to a local minimum of a curve.

In the present study, the effects of the inclusion of the instrument's dynamic behaviour on the minimum threshold pressure is focused on, so as to enrich the results given by the Linear Stability Analysis.

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