Broadband parametric amplification for nonlinear micro ring gyroscopes

Ahmed A.Barakat[*†‡], Peter Hagedorn[*]
[*]*Dynamics and Vibrations Group, fnb, TU Darmstadt, Darmstadt, Germany*
[†]*Graduate School of Computational Engineering, TU Darmstadt, Darmstadt, Germany*
[‡]*Design and Production Engineering Dept., Ain Shams University, Cairo, Egypt*

Summary. Parametric amplifiers have been well known in the electronic industry as well as in micro electromechanical systems (MEMS). However, they are typically confined to amplify the signals of these systems at primary or secondary resonance frequencies. Parametric amplification can nonetheless be extended to be broadband under certain conditions and system parameters. To the best of the authors' knowledge, this is not yet applied in microsystems, although being highly promising especially for the industry of micro sensors and transducers. On the other hand, in the industry of rate gyroscopes, the micro-ring gyroscope is being developed for over a decade in order to reach the accuracy level of tactical or inertial grade gyroscopes, while maintaining relatively low production costs of MEMS. For that sake, parametric amplification was studied for micro-ring gyroscopes with different excitation methods. However, the idea of having "global" amplification was not yet applied. For this reason, this contribution aims at presenting a parametric excitation method for nonlinearly modeled micro-ring gyroscopes which can provide "global" improvement in the amplification and the performance for a broadband frequency spectrum.

Introduction

Parametric resonances have long benn studied since the 1831 by Faraday. They were up to this date in practice the only useful phenomenon acquired through parametric excitation, i.e. through having a time-varying coefficient in the system's partial differential equation (PDE). The common practice, at least in the microsystems industry, did not make use so far from an important observation by Cesari in 1939. He found in at least two degree-of-freedom (DOF) systems, that an asynchronous parametric excitation can lead to "global" phenomena across the whole frequency spectrum [1]. This will be explained in the next section.

On the other hand, researchers aim at improving micro gyroscopes to attain the tactical grade level (0.1 deg/h bias stability), seeking the level of inertial grade (<0.01 deg/h) afterwards. For micro gyroscopes, nonlinear resonances were used in the literature to overcome asymmetries due to fabrication, as they allow energy transfer between vibration modes [2], thus improving sensitivity. Moreover, parametric resonances proved to "squeeze" mechanical and electrical noises [3], which contribute in improving the Angle Random Walk (ARW) and bias stability. Also by operating in the nonlinear regime, parametric excitations were found to provide much higher amplification, however, this was not adequately theoretically investigated till recently [4]. To this end, this work aims at extending the authors' investigation of the global effects induced by phase-shifted parametric excitations in linearly modeled micro-ring gyroscopes [5] to the nonlinear case.

Global effects

The previously mentioned global effects are discussed here for a linear system. In the current practice of paramteric excitation mechanisms, instability or amplification were exhibited at the primary resonance frequency equal to double the natural frequency, i.e. $\Omega = 2\omega_i$, or at the countably infinite secondary frequencies $\frac{\omega_i \pm \omega_j}{n}$, where $i = 1, 2,\ j = 1, 2,\ n \,\epsilon\, \mathbb{N}$. But by applying the theory of Cesari we could get an amplification and/or instability at every excitation frequency, hence, that is named as a "global" effect. One such consequence can be driving the system in a state of "total instability", i.e. at every frequency. Having said that, it may seem as bad news to engineering applications, however, for a system that behaves nonlinearly this could be translated to having higher amplitude, gain or amplification at a non-trivial stable stationary solution. This can be achieved by having phase-shifted off-diagonal time periodic functions in the parametric excitation matrix $\boldsymbol{C}(t)$. This effect can be simply illustrated for a two DOF system

$$\ddot{\boldsymbol{q}} + (\boldsymbol{D} + \boldsymbol{G})\dot{\boldsymbol{q}} + (\boldsymbol{K} + \boldsymbol{C}(t))\boldsymbol{q} = \boldsymbol{0}, \tag{1}$$

where $\boldsymbol{D}, \boldsymbol{G}, \boldsymbol{K}$ and $\boldsymbol{C}(t)$ are the damping, gyroscopic, stiffness and parametric excitation matrices, respectively. The resulting phenomenon can be illustrated in Fig.1. The figure shows a global destabilization of the system in absence of other excitation mechanisms, explained by having a positive eigenvalue at all frequencies except for *anti-resonances*.

Modeling of the nonlinear micro ring gyroscope

As shown in Fig. 2, the micro ring gyroscope is modeled as an elastic inextensible ring totally surrounded by curved electrodes, each at an angle $\bar{\theta}_k$. The ring was found to exhibit two degenerate modes at the first vibration frequency, the antinodes for the first mode and second modes are at angles $\bar{\theta}_k = (\frac{n\pi}{2}), (\frac{n\pi}{2} + \frac{\pi}{4})$ respectively. For modeling the ring's elastic dynamics we follow Natsiavas [7] by considering only flexural vibrations in thin rings. The elastic and electrostatic potential energy, U_d and U_e respectively, are then found to be

$$U_d = \frac{EI}{2R^3} \int_0^{2\pi} \left[u + \frac{\partial^2 u}{\partial \theta^2} + \frac{1}{2R} \left(\frac{\partial u}{\partial \theta} \right)^2 \right]^2 d\theta, \qquad U_e \simeq \frac{-\epsilon_0 b R}{2d} \sum_{n=0}^{4} \int_0^{2\pi} (V_{DC} + V_{AC})^2 \frac{u^n}{d^n} d\theta,$$

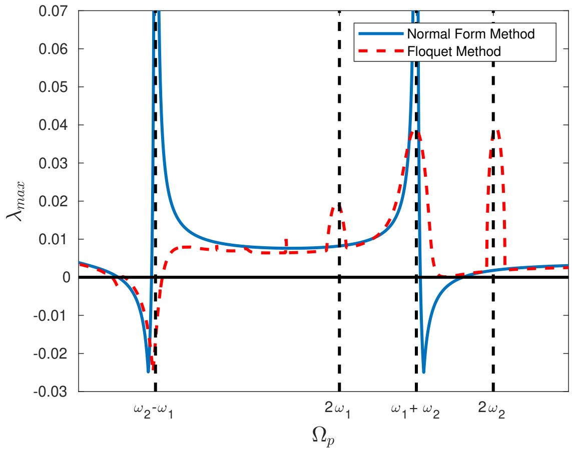

Figure 1: Global destabilization of a 2 DOF system

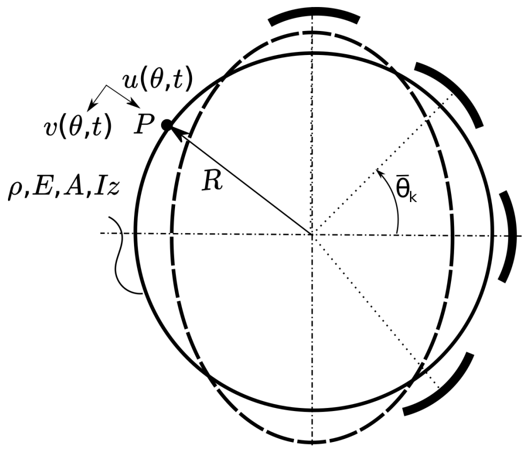

Figure 2: A sketch of the micro ring gyroscope

where R is the mean radius, u and v are the radial and circumferential displacements of a point on the centroidal axis respectively (s. Fig. 2), ϵ_0 is the electric constant, b, h are the ring's thickness and width respectively, d is the initial gap between each electrode and the ring's body, and V_{DC}, V_{AC} are the direct and alternating voltages respectively applied on the electrodes. On the other hand, the kinetic energy for the ring in a rotating frame will be

$$T = \frac{\rho bh}{2} \int_0^{d\pi} [u, v]^T . [u, v]^T R d\theta.$$

Applying the *ansatz* proposed by Natsiavas but for only two vibration modes, $u(\theta, t) = A(t)\cos(2\theta) + B(t)\sin(2\theta) - \frac{9}{16R}\left[A^2(t) + B^2(t)\right]$, and considering the alternating voltage to be distributed approximately according to $V_{AC}(\theta, t) = V_{AC}(t)\cos(2\theta)$, we get two nonlinear ordinary differential equations in $A(t)$ and $B(t)$ representing the system dynamics for the drive (primary) and sense (secondary) modes respectively.

Normal form transformation

However, since the resulting equations of motions are highly nonlinear, an analytical method is required to develop a better understanding of the gyroscope's dynamics. Different perturbation methods are widely used to analyze those systems, but we prefer here to use the normal form method, since it proved to have comparatively better accuracy for highly nonlinear systems involving parametric excitation. An approximate analytical solution is therefore derived, through which the highest eigenvalue could be calculated in terms of system's and excitation parameters, this was used as an indication for system stability. This can be seen in Fig. 1, in which the normal form method was used to simulate the eigenvalues of the system in (1) for all but resonant frequencies, complying with the numerical results calculated by the Floquet method. Two main findings are specially important; firstly the system is destabilized for approximately the whole frequency spectrum, that is specially important in tuning the exciation frequency. Since this would overcome the problem of the loss of sensitivity by mistuning. And secondly, this can give a new method of controlling the gyroscope's response through frequency control. This method will then be applied on the already derived equations of motion, and thereby the effect of the parametric excitation on the nonlinear micro gyroscope will be illustrated, in a similar way to what is explained for the linear system in (1).

Conclusions

This work contributes to understanding the observed nonlinear behavior of the micro-ring gyroscopes, since its nonlinear model was not sufficiently discussed in the literature [4]. Moreover, we propose a novel excitation method in the field of micro sensors, which offers an unprecedented flexibility in tuning as well as amplifying the micro ring gyroscope. This is believed to contribute to the development of current MEMS gyroscopes towards achieving an inertial grade accuracy.

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