Nonlinear modes of cantilever beams at extreme amplitudes: numerical computation and experiments

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Summary. Flexible, one-dimensional structures such as slender beams and cables are capable of undergoing highly nonlinear vibrations in extremely large amplitudes. A novel method for computing the behavior and, in particular, the nonlinear modes is here presented based on a geometrically exact finite element beam model that is solved using the continuation software MANLAB. An experimental setup is also described which utilizes a vibration table and Digital Image Correlation to physically observe the extreme amplitude vibration of the flexible cantilever beam.

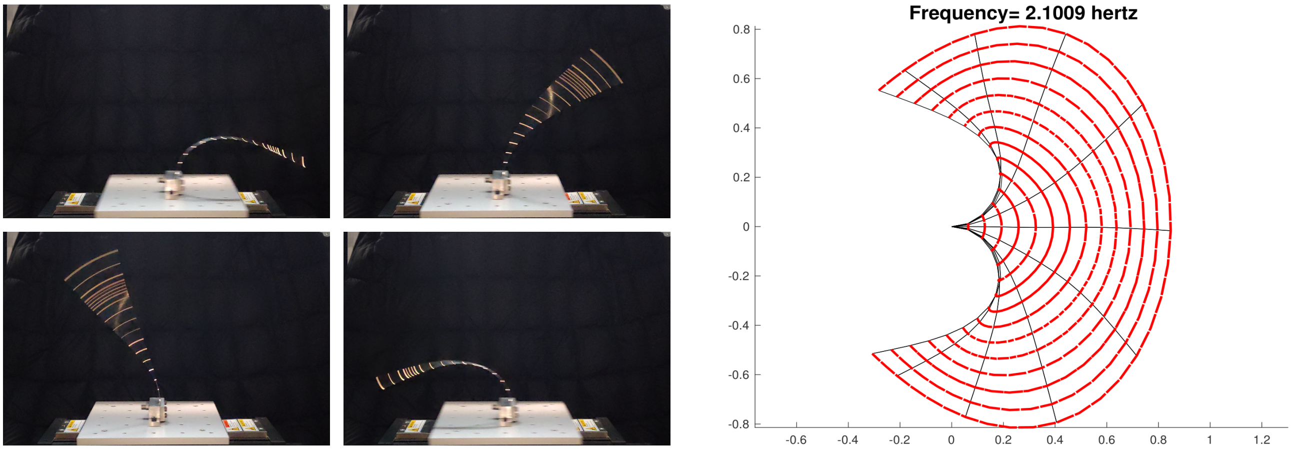

Figure 1 (Left): Four snapshots of preliminary experiments depicting the first nonlinear mode of a cantilever beam in extreme amplitude, achievable due to the bending capabilities of the flexible structure. Figure 1 (Right): Numerical computation of the first nonlinear mode of the cantilever beam, obtained using the path-following continuation solver, MANLAB [2, 3, 4].

Introduction

Although the field of nonlinear dynamics is vast and dense, nonlinear models of highly flexible structures are a subject of continuing research. Wires, cables, rods and hoses are examples of such flexible structures, as their slender, one-dimensional geometry and corresponding mechanical stiffness allow for extreme bending capabilities, especially when resonance conditions are considered. While several methods already exist for the computation of flexible systems, they are often limited, due to either the amount of required computation time or the limited range of applicability. Current strategies for simulation of nonlinear dynamics often utilize direct time integration schemes to solve the equations of motion. While often adequate for many purposes, these strategies can potentially encounter certain limitations, such as in detecting multiple solutions (if present) or bifurcation points. Additionally, further extensions to existing models are needed to capture the behavior of highly nonlinear dynamical systems, such as the present case of extreme amplitude vibration. For this reason, we here propose a custom continuation algorithm for the periodic solution of geometrically exact finite element beam models undergoing extreme amplitude vibrations. Numerical results are then compared with vibration experiments involving slender beam specimens as seen in Fig. 1 (Left).

Numerical model

A geometrically exact finite element discretization is performed for a cantilever beam geometry using Timoshenko-theory beam elements and a total Lagrangian nonlinear formulation [1]. The model is derived from the weak form variational formulation of the equation of motion of a cantilever beam rooted in the principle of virtual work and is traditionally written, for all time *t*,

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{f_{int}}(\mathbf{q}) = \mathbf{f_{ext}} \tag{1}$$

where $\mathbf{q}$ is a vector of size $3N$ that gathers nodal displacements u_i, w_i and θ_i representing the axial displacement, transverse displacement and cross section rotation of the ith finite element node, respectively, for N nodes: $\mathbf{q} = [u_1 w_1 \theta_1 \ldots u_N w_N \theta_N]^T$. $\mathbf{M}$ and $\mathbf{D}$ are the mass and damping matrices, respectively, each of size $3N \times 3N$; $\mathbf{f_{ext}}$ is the vector of applied external forces and $\mathbf{f_{int}}(\mathbf{q})$ is the nonlinear internal force vector, both of length $3N$, which is a nonlinear function of $\mathbf{q}$ based on the geometrical nonlinearities. The details of the finite element model can be found in Appendix 2 of [1].

Numerical solution

The geometrically exact finite element model described in (1) is then solved numerically using MANLAB, an interactive path-following solver rooted in MATLAB. Given harmonic forcing imposed on the cantilever beam, the resulting motion of the structure is sought periodic. The MANLAB solver employs a unique continuation scheme for periodic solutions, combining the Harmonic Balance Method (HBM) with the Asymptotic Numerical Method (ANM) continuation technique [2]. Traditionally, the principle of the HBM is to decompose a periodic function into a Fourier series truncated to a certain number of harmonics. However, in the present case of a geometrically exact cantilever beam, sine and cosine nonlinearities of the degrees of freedom are involved in the system. To reconcile these nonlinearities and others that may be present in any original system, the solution technique implemented in MANLAB proposes a quadratic recast of the original equation into a new system containing only polynomial nonlinearities of quadratic order or less. Following the quadratic recast, the derivation of the corresponding algebraic system for the Fourier coefficients is rather straightforward [2, 3]. The ANM continuation technique, which uses a pseudo-arc length parameterization, is then applied to yield the path-following solution that is visualized in MANLAB [4].

The combination of HBM and ANM has been shown to be both highly efficient and yet applicable to a broad set of systems containing non-polynomial nonlinearities [3, 4]. Recently, extensions to the method and, therefore, the MANLAB solver, allow for the stability analysis of the various branches of the periodic solution [4]. Using this solver, the computation time for the solution to (1) is greatly reduced and enables a numerical solution for the cantilever structure subject to extreme amplitude vibration (see Fig. 1 (Right)). In the case under study, the nonlinear modes of (1) are computed with periodic solutions of the unforced system, with $\mathbf{f_{ext}} = 0$, as explained in [4].

Experiments

In order to compare with the aforementioned numerical simulations, an experimental strategy has been designed to recreate the extreme amplitude vibrations of flexible specimens using a Long Stroke Shaker vibration table. The experimental strategy here described utilizes a Phase-Locked Loop (PLL) controller, which, for a harmonic excitation $F(t) = F \sin(\Omega t + \phi)$, changes the frequency Ω so that a given phase lag between the phase ϕ of the force signal and that of the system response is induced [5]. Adjustment of the amplitude of excitation with the desired phase lag set to $\pi/2$ provides a robust method for measuring the backbone curve of the system, equivalent to its nonlinear modes. A long and thin stainless steel beam is prepared for the experiments and is fixed at the base to the vibration table to recreate the cantilever condition under investigation. The table vibrates at the selected amplitude F and frequency Ω, thereby exciting the cantilever beam specimen. Preliminary experiments are able to demonstrate the extreme amplitude vibrations using reflective points along the beam (Fig. 1 (Left)). The displacement field along the beam is measured based on differences in the illuminated points through a series of images captured with a rapid motion camera in a technique known as Digital Image Correlation (DIC). The results of the DIC are then compared with numerical results obtained from MANLAB.

Conclusions

In this paper, numerical and experimental extensions to existing strategies for calculation of nonlinear modes of cantilever beams in large amplitude vibrations are presented. A geometrically exact finite element numerical model of the equation of motion is developed and implemented into the MANLAB continuation solver for periodic solutions. The solver is able to describe the motion of highly flexible structures even at extreme amplitudes, which is then compared with experimental results obtained from vibration tests integrating a Phase-Locked Loop for robust calculation of the nonlinear modes. In conclusion, the procedure outlined in this paper presents a novel and efficient method for the calculation and simulation of highly geometrically nonlinear beam structures, even at extreme amplitudes of vibration.

References

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