# Parametric analysis of a Nonlinear Energy Sink for an unstable dynamic system

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*Summary*. The theoretical study of the change of steady state regime of an unstable dynamic system coupled to a nonlinear energy sink with the respect of a set of parameters is developed in this paper. This study is carried out by the use of asymptotic methods (multiple scale method mixed with harmonic balance method), it leads to singular perturbed system that is studied with geometric singular perturbation theory. The steady state regime of the dynamic system is linked to the information extracted from the singular perturbed system : the slow flow fixed points and their stability. From this information, analytical conditions are established. These conditions lead to parametric analysis and bifurcation diagrams which are describing the mechanism of steady state regime change. This study will allow to test huge and important set of parameters for a NES rapidly in comparison of numerical or experimental studies.

## Introduction

The use of Nonlinear Energy Sink (NES) based on Targeted Energy Transfer is very efficient solutions for passive control. A NES is a device composed of a light mass or inertia compared to the main system, a nonlinear stiffness and a viscous linear damper. NES have been developed for the past ten years and many different technologies are used in order to realize them experimentally. A state of the art is presented in [1] about these technologies such as piece-wise linear, non-smooth, vibro-impact, bistable and magnetic NES.
Among several applications, NES are used to control dynamic instabilities or resonances, e.g.: helicopter ground resonance, friction system (Hultèn's model), airfoil system and systems under periodic excitation.
In order to control unstable or resonant system through nonlinear vibration absorber, a design approach needs to be carried out. The method used for designing a NES consists in predicting the response of the nonlinear system through asymptotic analytical approach. Among different approaches, the complex-averaging method (CX) [2] and the method of the multiple scale combined with the harmonic balance method (MMS-HBM) [3] have been extensively applied. These approaches lead into a singular perturbed set of differential equations of the modulation response amplitude.
This system of equation is later treated by applying the Geometric Singular Perturbation Theory (GSPT) [4] which decompose the complex motion of the system into a fast and a slow dynamic. Such methodology has been recently used to survey the response regimes reached with a NES coupled to a primary system [5]. Based on the best authors knowledge, the GSPT has only been used on the modulation equation obtained with CX-method and this approach has not been further developed for parametric study of the NES's parameters applied on the mitigation of unstable systems.
In this work, the design of NES used to mitigate divergent oscillations of unstable system is carried out. The fast and flow dynamics of the singular perturbed system obtained with the combined MMS-HBM is treated with GSPT. The relation between the critical manifold of the fast dynamics, the nature and position of the fixed points of the slow dynamics and the NES response is evaluated. Finally, an extensive parametric study maps the different regimes of the dynamic system for a large set of NES's parameters and instability severity of the primary system.

## Development and Results

The dynamic system considered is a single degree of freedom composed of a mass $M$, a linear stiffness $K$ and a linear negative damping coefficient $C$ (SDOF in black line) coupled to a NES, composed of a mass $m$, a nonlinear stiffness $k$ and a linear damping factor $c$ (red line) attached (see Figure 1).

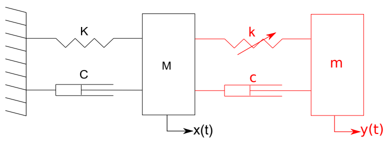



Figure 1: Mechanical model : in black an unstable SDOF, in red a Nonlinear Energy Sink (NES) attached.

By introducing a relative displacement $v = x - y$ between the main system and the NES and the usual parameters such as $\omega^2 = \frac{K}{M}$, $\zeta = \frac{C}{2\omega M}$, $\lambda = \frac{k}{M}$, $r_c = \frac{c}{M}$ and $r_m = \frac{m}{M}$ the equations of motion are obtained :

$$\begin{cases} \ddot{x} + 2\zeta\omega\dot{x} + \omega^2 x + r_c\dot{v} + \lambda v^3 = 0 \\ r_m\ddot{v} - r_m\ddot{x} - r_c\dot{v} - \lambda v^3 = 0 \end{cases} \quad (1)$$

After applying the whole method (MMS-HBM and GSPT) and getting the information from the slow flow (SIM, fixed points, singular folded points, slow flow dynamics and stability) and the fast flow (modulation equation and dynamic fixed point) it is possible to obtain the dynamic of the system represented in Figure 2.
The slow-flow fixed points in red are governing the response of the phase portrait (in black) Figure 2. The cycle is governed mainly by the SIM (in blue), after reaching the first fold point, the system has enough energy to jump on the

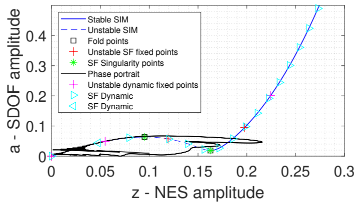
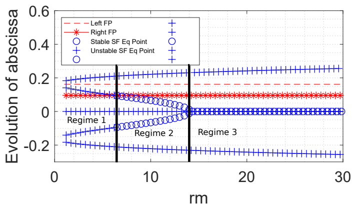
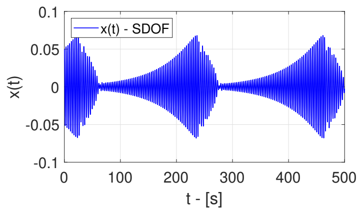



Figure 2: Nonlinear manifold, Slow flow information and Dynamic information with physical parameters $r_m = 4\,\%$, $\alpha = 2\,\%$, $\zeta = -1\,\%$, $\Lambda = 50$.

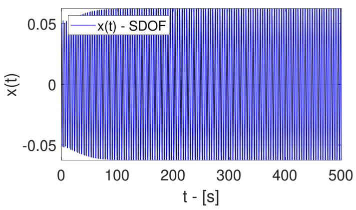
other stable branch of the Slow Invariant Manifold (SIM). The highest fixed point which is unstable is forcing the system's amplitude to decrease to the second folded point in order to jump back on the first stable branch of the SIM.
From a parametric study, it is possible to understand that the stability (highly linked to the position) of the slow flow fixed points governs the change of steady state response. It is also possible to highlight it by producing a bifurcation diagram (Figure 3a).



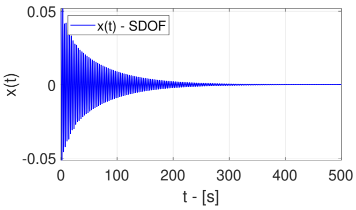

(a) Bifurcation diagram for $\alpha = 25\,\%$, $\zeta = -1\,\%$, $\Lambda = 50$.




(b) Regime one.




(c) Regime two.




(d) Regime three.

Figure 3: Bifurcation mechanism.

Figure 3a shows the mechanism responsible for the steady state regime change. There are three specific areas that will lead to three different regime. The regime one corresponds to a Limit Cycle Oscillations (LCO) with strong or low modulated response (Figure 3b). This regime appears when the intermediate slow flow fixed point is between the two folded singular point of the SIM and unstable. When this fixed point is moving toward the first folded singular point, it becomes stable and then leads to the regime two (Figure 3c). Finally, this point move to the static fixed point (0,0) and leads to the regime three (Figure 3d).

## Conclusion

The analytical study of a complex dynamic system composed of an unstable single degree of freedom and a NES through asymptotic methods and singular perturbation method have been addressed. The theoretical result obtained allowed to improve the influence of the NES parameter on the steady state regime of the dynamic system by conducting extensive parametric study and computing bifurcation diagrams. All these results will now be used to design a NES theoretically and reduce the numbers of iterations needed to design a NES that will mitigate the response of an unstable dynamic system.

## References

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