Determination of performance parameters of nonlinear galloping energy harvesters using Jacobi elliptic functions

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Summary. In the work, through analytical considerations, the peak efficiency of three different variants of galloping energy harvester was defined. For this purpose, the authorial method based on elliptic harmonic balance was employed, consisting of comparison of impossible to analyze, accurate high order solutions, and simplified solutions of a linearized model. Research has shown that the peak efficiency of the hardening and bistable devices is greater by 17% and 30% respectively in regards to the linear device, while application of softening stiffness always leads to a loss of efficiency.

Introduction

In the era of the idea of the Internet of Things, the desire of scientists, engineers, medics, and even not-professionals is to continuously measure countless physical phenomena that occur both in our surroundings and at great distances beyond direct human reach. This increases the requirements for measuring devices and thus for their power supply – if access to the operating device is limited, it may not be possible to route the power cables or periodically exchange the batteries. The solution to this problem may be the application of autonomous devices - equipped with their generator harvesting ambient energy. An example of such a generator is the galloping energy harvester (GEH) - the device that allows to harvest the energy of vibrations induced by the flow.

In its simplest version, the GEH can be considered as a body (resonator) mounted on the elastic element, coupled to the piezoelectric (Fig. 1.), described by the mathematical model (1) – (2). If an appropriately shaped body is used, the geometry of which is represented by the coefficients a_1 and a_3, at a certain flow velocity, called the critical velocity, negative damping will be induced in the system and thus stability of the system will be lost.

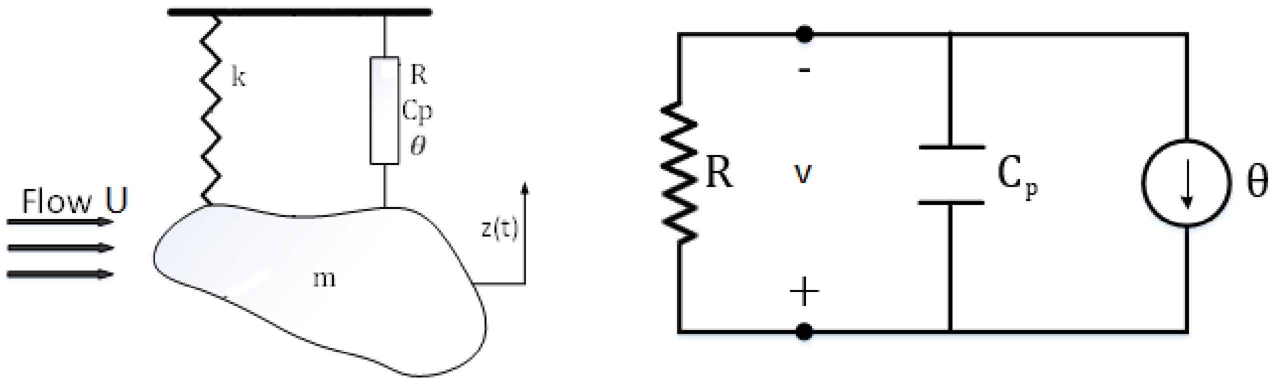

Figure 1: Model of the aeroelastic energy harvester

$$m\,\ddot{z}(t) + S\big(z(t)\big) - \theta\, v(t) = -\frac{1}{2}\rho h\left(a_1\, U\, \dot{z}(t) + a_3\frac{\dot{z}(t)^3}{U}\right) \tag{1}$$

$$C_p\dot{v}(t) + \frac{v(t)}{R} + \theta\dot{z}(t) = 0 \tag{2}$$

where: m – mass of the body, $S(z(t)) = k\, z(t)$ – restoring force, $z(t)$ – displacement, $\dot{()}$ and $\ddot{()}$ – first and second time derivative, h – characteristic length of the body, ρ – fluid density, U – flow velocity, a_1, a_3 – experimentally determined coefficients, $v(t)$ – voltage, θ – piezoelectric coefficient, R – circuit resistance, C_p – circuit equivalent capacity.

The efficiency of the device

One of the most important parameters describing energy generators is peak efficiency η_{peak}. In [1], the maximum efficiency was derived for the simplified linear GEH model, in which the harvested was defined as structural damping. In work [2], we confirm the validity of the obtained results also for the full electromechanical model. These results indicate that the peak efficiency of such a device depends only on the geometry of the resonator and has a form:

$$\eta_{peak} = -\frac{{a_1}^2}{6\,a_3} \tag{3}$$

The natural path of exploring this matter are studies of the possibilities offered by the application of a system with different types of non-linear stiffness characteristics. Although there already studies that compare different GEH variants, e.g. [3], but none of them have defined the efficiency of the devices.
Our earlier studies proved that the analytically formulated limit cycle of the model of the device can be obtained through the harmonic balance assuming some form of the model solution. In the case of devices with a more complex structure, the solutions obtained with this method are, however, too complicated to draw appropriate conclusions from them. Preliminary studies indicate that an excellent alternative may be the elliptic harmonics balance method, where the solution of the model is assumed in the form of elliptic Jacobi functions. Because of that, in the paper, we propose a method of solving this problem based on a comparison of a linearized, easy-to-analyze solution and a very precise one, obtained with elliptic harmonic balance method. The analytical forms of both solutions are very similar, and the value of the factors by which they differ can be strictly defined at extreme points.

Results and conclusions

Employing the elliptic harmonics balance, the expressions describing the efficiency of various variants of devices were obtained as a function of the flow velocity in the form of $\eta_N = \eta_L\,\Psi$, where $\eta_L = \eta_L(U)$ is the efficiency of linear device and $\Psi = \Psi(m)$ (Fig. 2) is the coefficient that describes the impact of nonlinearity on the efficiency of the device in function of the modulus of elliptic function m. Depending on the nature of the nonlinearity, the value modulus is bounded in the following ranges: for hardening stiffness $0 < m < 0.5$, for softening stiffness $0 < m < 1$ and for bistable system $0.5 < m < 1$, it is therefore, possible to strictly determine the value of the Ψ coefficient at extreme its extreme values.

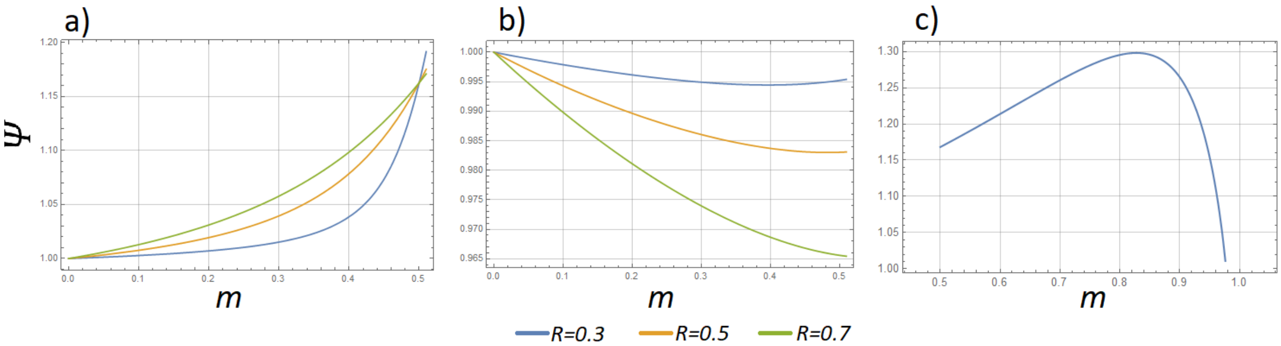

Figure 2: Ψ values as a function of m for different R values and different stiffness variants: a) hardening, b) softening, c) bistable

Based on the above information, it can be concluded that: a) the function $\Psi(m)$ for the system with hardening stiffness depends on the values of the system parameters, but for the $m = 0.5$ it always has the same, maximum value $\Psi(0.5) \approx 1.17$, b) maximum value of $\Psi(m)$ for the system with softening stiffness is reached for $m = 0$ and $\Psi(0) = 1$ therefore, the softening stiffness will lead to a decrease in peak efficiency, c) regardless of the system parameters, the peak value of $\Psi(m)$ for the bistable system is $\Psi(0.83) \approx 1.30$.

References

[1] Barrero-Gil, A., Alonso, G., and Sanz-Andrés, A. (2010) Energy harvesting from transverse galloping. *Journal of Sound and Vibration*, 329(14), 2873– 2883.
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[3] Bibo A., Alhadidi A. H., Daqaqa, M. F. (2015) Exploiting a nonlinear restoring force to improve the performance of flow energy harvesters, *Journal of Applied Physics*, 117.