Explanation of the Locomotion of a Rigid Body along a Vibrating Nonlinear Beam

Florian Müller*, Alexander F. Vakakis‡ and Malte Krack*
University of Stuttgart, Germany
‡*University of Illinois at Urbana-Champaign, USA*

Summary. Several independent research groups have investigated the self-adaptive behavior of a clamped-clamped beam with attached slider. Under harmonic base excitation the slider passively moves to a certain position on the beam, going along with a significant increase of amplitude. We are the first to fully explain this complex process theoretically by means of a model reproducing the behavior. Moreover, we show that the slider's movement takes places on a slower time scale compared to the beam's vibration. Exploiting this temporal separability, we explain the different mechanisms yielding transport towards or away from beam's center.

Introduction

A harmonically forced clamped-clamped beam with a slider free to move axially on the beam has been experimentally investigated by different groups [1, 2, 5]. After initially small vibrations, the slider moved to a certain position, where vibration amplitude increased significantly. After this process, the amplitude and the final slider position, which is in many cases away from the antinode at beam's center, were maintained in steady state. In case the slider is initially placed close enough to the beam's center, it moves towards clamping first and turns back towards center at some point. The turning point is accompanied with a jump to higher vibration level and the movement back goes along with a further increase of amplitude [1]. In [1, 2], beside experimental results, models were presented, assuming that the slider is constrained to the beam in the vertical direction. Simulation results exhibit a slider movement towards the beam's center going along with an increase of amplitude. However, a movement towards clamping, a jump of amplitude and a steady state slider position away from center couldn't be reproduced. In [3, 4], we present a model accounting for a gap between slider and beam, which gives rise to unilateral and frictional contact interactions and consider also the beam's stiffening geometric nonlinearity. This model has the capability to reproduce all features of the self-adaptive process. In [5] we present an experimental validation of this model, yielding excellent agreement of measurements and simulation. In the present contribution we demonstrate that the beam's vibration and the slider's movement take place on well separated time scales and exploit this separability to study the different mechanisms producing locomotion of the slider systematically.

Separable timescales of vibration and slider movement

The considered specification of the self-adaptive beam-slider system corresponds to the beryllium-copper beam described in [1]. A schematic of the model is depicted in Fig. 1a. The beam is modeled according to Euler-Bernoulli theory, where the deformation is approximated by the lowest frequency bending mode and the stiffening geometric nonlinearity due to the fixed ends is considered. To model unilateral and dry frictional contact interactions between beam and slider, the Signorini and Coulomb laws combined with Newton's impact law are used. The resulting set-valued force laws are solved numerically applying Moreau's time stepping scheme. The contact model and simulation procedure are adopted from [3]. A simulation of a successful self-adaptive process is depicted in Fig. 1b in terms of the vibration's amplitude as a function of the resulting slider position (dashed black). While vibration amplitude is initially small, the slider moves towards clamping. At $\tilde{s}/L \approx 0.24$, the slider turns and the amplitude jumps to a higher level. After this, the slider moves back towards center, while the amplitude increases further. During the whole process, the global horizontal movement of the slider is slow compared to the beam's vibration [4]. To prove the independence of these time scales, we study the steady state response depending on the relative slider position. In order not to affect the slider's dynamics by an additional constraint, we let the slider's horizontal displacement s free, but consider the given relative slider position $\tilde{s}$ always when evaluating the contact kinematics during the simulation. This essentially yields a model where the beam is moved with the slider horizontally [5]. For each relative slider position we numerically integrate until steady state, increase the relative slider position by a small $\Delta\tilde{s}$ and again integrate until steady state, starting from the final state of the previous point. The results for increasing and decreasing $\tilde{s}$ exhibit a high and a low amplitude branch, see Fig. 1b (green and orange lines). This behavior can be explained by the beam's stiffening nonlinearity [4]. Clearly, the response of the system with free slider follows the low amplitude branch first, jumps, and follows the high amplitude branch then. The steady state model enables us to evaluate also the slider's absolute horizontal displacement s. In Fig. 1c we illustrate the steady state absolute slider velocity by means of slider's displacement per beam length and evaluation periods. In the range of $\tilde{s}$ which is relevant for the self-adaptive process, the steady state slider velocity corresponding to the low (high) amplitude branch points towards clamping (beam's center). This is in qualitative and good quantitative agreement with the self-adaptive process. Also on the fast time scale of vibration, the steady state model coincides very well with the system with free slider (time histories not shown for brevity). Alltogether, this proves the separability of the different time scales [6].

Vibration induced locomotion towards center or clamping

We now want to explain, why the slider is transported towards clamping (beam's center) during the low (high) amplitude phase. To this end, we study the periodically dominated vibrations obtained by the steady state model exemplary

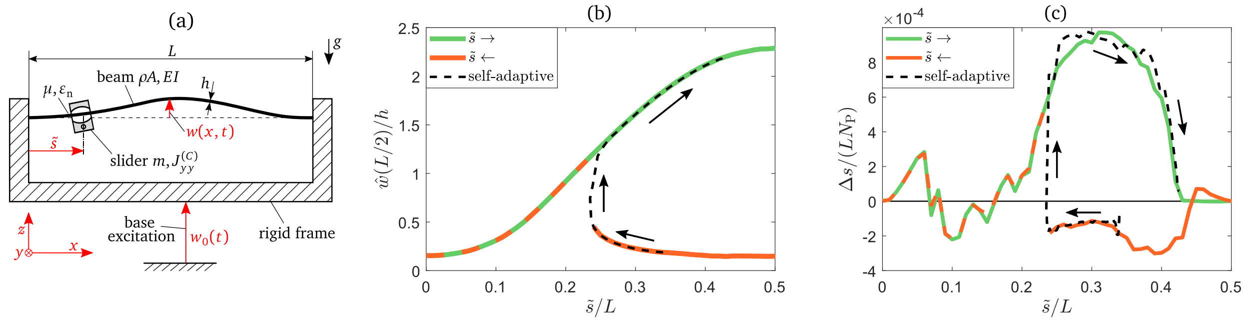

Figure 1: (a) Two dimensional model of self-adaptive system. (b) Vibrational amplitude versus relative slider position. (c) Averaged normalized slider velocity versus relative slider position. [6]

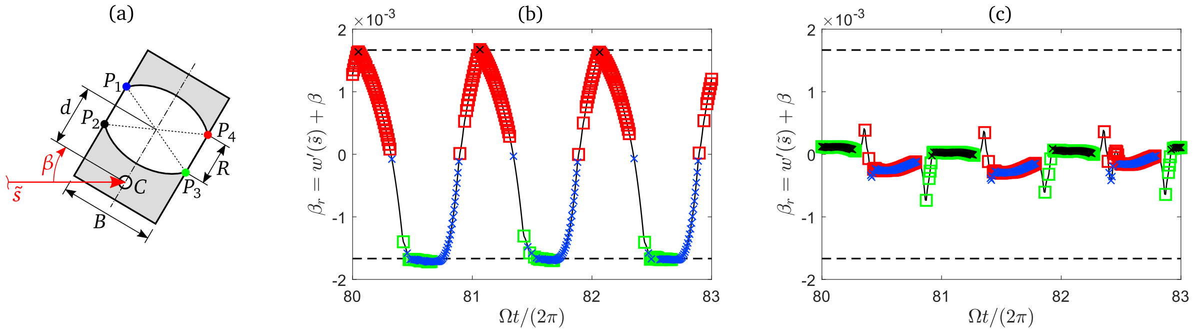

Figure 2: (a) Slider detail. (b) Relative slider rotation (steady state model, $\tilde{s} = 0.3$, low vibration level). (c) Relative slider rotation (steady state model, $\tilde{s} = 0.3$, high vibration level). Markers indicate active contacts, where the colors match with P_1 through P_4 in (a). [6]

at $\tilde{s}/L \approx 0.3$. We start with the simple explanation for the high amplitude phase. Here, the beam's elastic deformation is close to a monoharmonic oscillation which is approximately in phase with the excitation and big compared to the clearance between slider and beam. During almost one half of an excitation cycle, both upper contact points are in contact. During the other half of the cycle, both lower points are in contact, see Fig. 2c. Therefore the system behaves like a beam with vertically constrained but horizontally movable body. For this simplified system, the inertia force acting on the slider has a horizontal component stemming from the beam's slope, which can be approximated $m\ddot{s} = -m\left[\ddot{w}(s) + \ddot{w}_0\right] w'(s)$. This force points always to center. In case of a very small [3, 4] or no [2] gap, the slider moves to center also for low amplitude level. Instead of that, successful adaptation is obtained in case the clearance is chosen in the order of magnitude of the vibration level corresponding to the low amplitude branch, which allows for a complex limit cycle, see Fig. 2b. This limit cycle contains states of maximum and minimum possible relative rotation between slider and beam. The crucial transportation of the slider's center of mass C towards clamping is obtained during rolling on the upper contacts P_1 and P_4 from minimum to maximum possible rotation [6].

Conclusions

The beam's vibration and slider's movement take place on well separated time scales, which can be shown by using a steady state model with prescribed relative slider position. Studying the periodically dominated steady state responses corresponding to the different phases of the self-adaptive process explains the slider's locomotion. Further work could focus on optimization of the system and application as energy harvester or vibration absorber.

References

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