A Nonlinear Piezoelectric Shunt Absorber with 2:1 Internal Resonance

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Summary. This abstract presents the design of a new nonlinear vibration absorber that aims to attenuate the vibration of an elastic structure near its resonance frequency. This absorber is formed by connecting the elastic structure via a piezoelectric patch to an electrical shunt circuit consisting of a resonant shunt (R-L circuit), combined in series with a quadratic voltage component. The 2DOF reduced order model showed that, for suitable tuning, a 2:1 internal resonance could be generated leading to a creation of an anti-resonance in the response of the elastic structure at the resonance frequency of the mode to be attenuated. The anti-resonance amplitude is shown to remain constant after a threshold excitation level giving this absorber an advantage over the linear absorbers.

Nonlinear Piezoelectric Shunt Absorber Design

The shunt absorber is designed by connecting an electric shunt circuit, consisting of a resistor R and an inductor L in series with a quadratic nonlinear voltage component, to an elastic structure through a piezoelectric patch, as shown in Fig. 1. The quadratic nonlinear term is set to be proportional to the square of the voltage across the piezoelectric patch V_p by a gain equals to β. By introducing this quadratic non-linearity in the shunt circuit and by tuning the electrical resonance

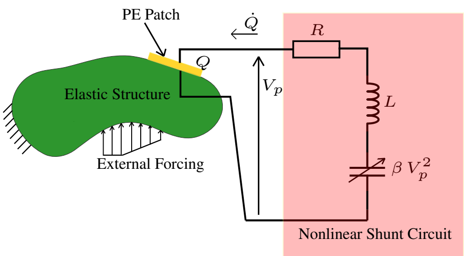

Figure 1: Nonlinear shunt circuit schematic

frequency to be at one half of the resonance frequency of the structure mode to be attenuated, a 2:1 internal resonance would occur. Such an internal resonance, when forcing above a threshold level, leads to an energy transfer from the mode to be attenuated to an electrical mode at the first sub-harmonic frequency. As a result, a kind of anti-resonance at the resonance frequency of the structure is created and the amplitude of the anti-resonance is shown to be independent of the excitation level, leading to a saturation phenomenon [1, 2].

Reduced Order Model and Numerical Results

The main equations governing an elastic system subjected to harmonic excitation and connected to a nonlinear resonant shunt circuit through a piezoelectric patch as shown in Fig. 1, are: (see [3])

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{D}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} + \mathbf{K}_c V_p = \mathbf{F}\cos(\Omega t)\,, \tag{1a}$$

$$C_p V_p - Q - \mathbf{K}_c \mathbf{u} = 0\,, \tag{1b}$$

$$V_p + L\ddot{Q} + R\dot{Q} + \beta V_p^2 = 0\,, \tag{1c}$$

where $\mathbf{M}$, $\mathbf{K}$, and $\mathbf{D}$ are the mass, stiffness, and damping matrices respectively. $\mathbf{K_c}$ is the electro-mechanical coupling coefficients vector, C_p is the piezoelectric patch capacitance, and $\mathbf{F}$ is the excitation force vector. A reduced order model is constructed by performing two modal expansions. The first one is obtained by expanding the displacement vector denoted by $\mathbf{u}$ on the basis of the N linear modes. This would lead to a linearly coupled 2DOF system in terms of the charge Q and the modal displacement q_i, by truncating the displacement at the i-th mode to be attenuated. Then, a second modal expansion is done, on a basis of two modes to obtain the following fully quadratic differential system:

$$\ddot{x_1} + 2\mu_1\dot{x_1} + \omega_1^2 x_1 + \Lambda_1 x_1^2 + \Lambda_2 x_1 x_2 + \Lambda_3 x_2^2 = f_1 \cos\Omega t\,, \tag{2a}$$

$$\ddot{x_2} + 2\mu_2\dot{x_2} + \omega_2^2 x_2 + \Lambda_4 x_1^2 + \Lambda_5 x_1 x_2 + \Lambda_6 x_2^2 = f_2 \cos\Omega t\,. \tag{2b}$$

Eq. (2) is solved using the Multiple Scales Method (MSM) to have a closed form solutions that are used to study the effect of each design parameter on the response, so an optimized parameters selection could be established to achieve the highest attenuation. In addition, the Harmonic Balance Method (HBM), coupled with a numerical continuation procedure based on the Asymptotic Numeric Method (ANM) [4, 5], is used for results validation. Then, the results could be transferred back to the physical coordinates, which are plotted in Fig. (2). It can be observed that the stable solution of q_i stems from the principal resonance curve through a branching point bifurcation, reaching a minimum at the resonance frequency.

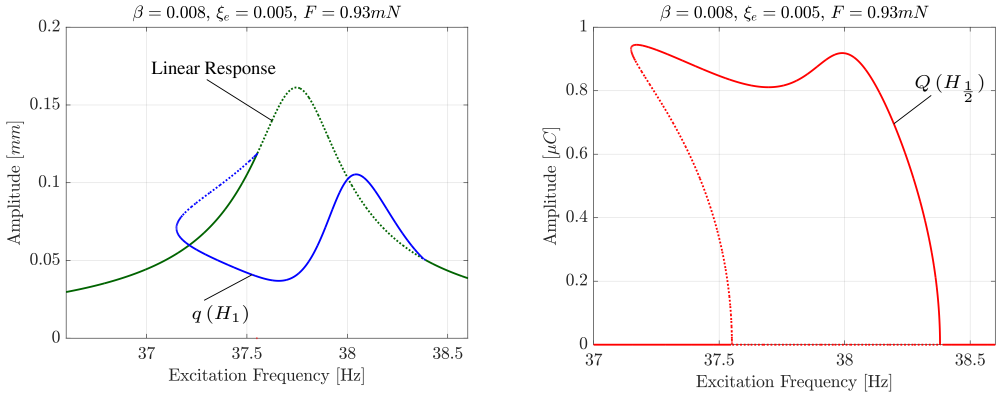

Figure 2: Numerical Results of the amplitudes of q_i (left) and Q (right) based on HBM. Solid line for stable solution and dotted line for unstable solution.

Experimental Setup and Results

Experimental tests are conducted on a cantilever beam to attenuate the first bending mode by connecting the nonlinear shunt circuit through a piezoelectric patch glued at the beam bottom. The excitation is done on the beam tip by inducing a current in a coil-magnet system, as shown in Fig. 3. The nonlinear shunt circuit consisted of an Antoniou synthetic inductor, to achieve a high inductance value, in series with a multiplier circuit composed of a set of Op-Amps and a signal

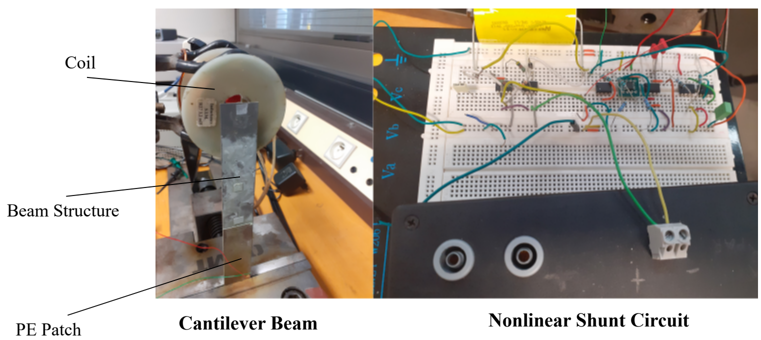

Figure 3: Experiment Setup

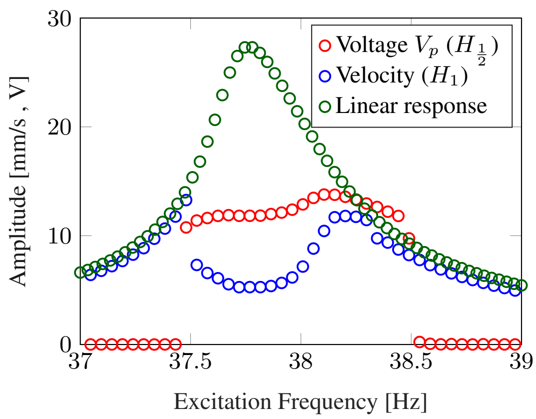

Figure 4: Experimental Results

multiplier, to generate the quadratic voltage behaviour. The time signal of the velocity is measured also at the beam tip using a laser vibrometer, and the voltage across the piezoelectric patch is measured using a voltage probe, which is then fed back to the input of the multiplier circuit. The harmonics amplitude could be extracted from the time signal, and the experimental results are shown in Fig. 4, which clearly validate the numerical results in Fig. 2.

Conclusions

In this study, a semi-passive nonlinear piezoelectric shunt absorber is designed for the first time to attenuate the vibration level using the 2:1 internal resonance features. A 2DOF reduced order model has been constructed, in which the numerical and experimental results appeared to be in full accordance. In addition, when connecting the absorber to a cantilever beam, a high attenuation is achieved near the resonance frequency of the first bending mode.

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