Experimental Verification of Stability Theory for a Planar Rigid Body with Two Unilateral Frictional Contacts

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Summary. Stability of equilibrium states in mechanical systems with multiple unilateral frictional contacts is an important practical requirement, with high relevance for robotic applications. In our previous work, we theoretically analyzed finite-time Lyapunov stability for a minimal model of planar rigid body with two frictional point contacts. Assuming inelastic impacts and Coulomb friction, conditions for stability and instability of an equilibrium configuration have been derived. In this work, we present for the first time an experimental demonstration of this stability theory, using a variable-structure rigid "biped" with frictional footpads on an inclined plane. By changing the biped's center-of-mass location, we attain different equilibrium states, which respond to small perturbations by divergence or convergence, showing remarkable agreement with the predictions of the stability theory. Using high-speed recording of video movies, good quantitative agreement between experiments and numerical simulations is obtained, and limitations of the rigid-body model and inelastic impact assumptions are also studied. The results prove the utility and practical value of our stability theory.

Many robotic systems are based on establishing contacts between bodies, for performing tasks of object manipulation or locomotion. Several characteristic types of contact-based robotic motion exist. In robotic grasping, contacts are typically used to enforce kinematic constraints that immobilize an object with zero relative motion, so contacts are maintained persistent. On the other hand, dynamic tasks such as object juggling and rapid legged locomotion involve intermittent contacts where impacts induce non-smooth transitions in contact states. An intermediate regime uses *quasistatic* manipulation and locomotion tasks with non-prehensile contacts. Such motion often relies on unilateral contacts that are maintained in persistent no-slip state imposed by equilibrating forces that satisfy frictional contact constraints. A common example is quasistatic legged locomotion on rough terrain where gravitational load is resisted by contact forces at the feet's supports.

In the regime of quasistatic motion with unilateral contacts, it is of practical importance to consider *stability* of multi-contact equilibrium states under disturbances caused by model uncertainties, joint coordination inaccuracies, irregular contact surfaces, and more. Common approaches consider robustness of the solution for equilibrium contact forces under disturbances such as localized elastic deformations at contacts or margins of potential energy. A main limitation of these approaches is assuming persistent contacts without accounting for dynamics under small initial perturbations about equilibrium, that do not necessarily maintain contact constraints. This is close in spirit to the well-known concept of *Lyapunov stability* in dynamical systems theory. Analysis of this type of dynamic stability in multi-contact systems is challenging, since any small initial perturbation of displacements and velocities immediately induce response governed by hybrid dynamics involving non-smooth transitions between contact states and impacts. Such systems often involve complicated phenomena such as solutions with *Zeno behavior* and more rarely, *Painlevé paradox* where friction-dominated solution is either indeterminate or inconsistent.

Our recent joint work [1] presented theoretical analysis of finite-time Lyapunov stability for a planar rigid body with two frictional contacts and frictional inelastic impacts. The analysis in [1] reduced the hybrid dynamics of the system in close vicinity of an equilibrium state to a scalar \Poincare map R and scalar magnitude-growth function G, which together encompass the entire response and contact state transitions. Under specific restrictions called *persistent equilibrium*, the work [1] derived theoretical conditions for stability and instability of frictional two-contact equilibria based on properties of the semi-analytic functions R and G, and showed how stability can depend on structural parameters such as friction coefficients and center-of-mass location relative to contact positions.

The goal of this work is to present, for the first time, an experimental demonstration of our stability theory from [1]. Our experimental setup consists of a rigid "biped" with variable structure, which is perturbed from frictional two-contact equilibrium state on an inclined plane. We first present extension of our theoretical analysis in [1] to account for a relaxed notion called *weakly-persistent equilibria* and also derive a simpler stability condition. Both modifications cover cases which are relevant to actual properties of our experimental biped and contact geometry. Upon shifting the biped's variable center-of-mass, our theoretical predictions indicate changes between the two instability mechanisms towards stability. These stability transitions are demonstrated experimentally, and high-speed camera recording enables tracking the biped's motion for quantitative comparison with theoretical simulations, as well as assessing the validity of our rigid-body model assumptions. We find excellent qualitative and good quantitative agreement between the theoretical predictions and experimental measurements, and conclude that our model slightly underestimates stability, where discrepancies are mainly due to added energy dissipation caused by damped elastic vibrations and footpads' compression during impacts. The results demonstrates the utility and practical value of our stability theory. Our present work is based on our recently submitted paper [2].

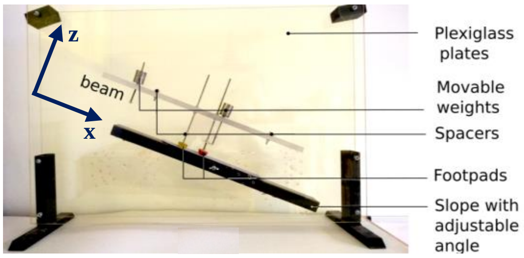

Figure 1: Our experimental setup of a biped on a slope, with variable center-of-mass (COM) and unequal friction on both footpads. The biped is given slight initial perturbation from equilibrium and its response is a sequence of impacts and contact states. Stability and instability can be changed by shifting the biped's COM.

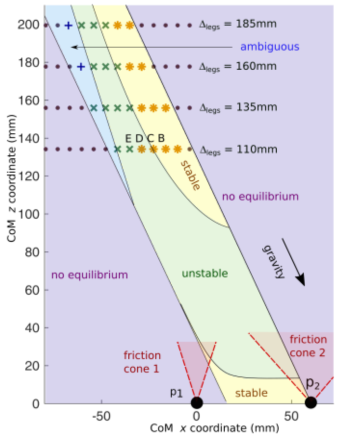

Figure 2: Plot of the biped's COM locations in (x,z) plane, where p1, p2 denote the footpads' contacts. Colored areas denote COM regions where our theoretical analysis predicts stability, instability, ambiguous equilibrium and no equilibrium. The grid of markers denote experimental results:
'●' – no equilibrium
'+' – ambiguous equilibrium
'x' – unstable equilibrium
'*' – stable equilibrium

References

[1] Varkonyi, P. L., & Or, Y. (2017). Lyapunov stability of a rigid body with two frictional contacts. *Nonlinear Dynamics*, *88*(1), 363-393.

[2] Or, Y., & Varkonyi, P. L. (2020). Experimental Verification of Stability Theory for a Planar Rigid Body with Two Unilateral Frictional Contacts, Submitted 2020. *Preprint at arXiv:2008.10323*. Video movie at this LINK.