Adiabatic phenomena in particle accelerators

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Summary. In the hadron colliders the transverse dynamics of a charge particle in a magnetic lattice is well modeled by a Hamiltonian formalism and the Poincaré map of the system is a polynomial symplectic map with a elliptic fixed point (reference orbit) in two or three degrees of freedom. The presence of fast-slow variables or slow modulations in the dynamics has been considered to estimate the dynamic aperture (i.e. the transverse phase space region that guarantees a long term stability of the orbits) or to perform the beam shaping in the synchroton motion. Recently, the possibility of slowly modulating the linear frequencies of the reference orbit (linear tunes) in presence of non-linear resonances in the phase space has been proved to provide a very efficient mechanism for beam extraction by means of the adiabatic resonance trapping phenomenon (Multiturn Extraction). The multiturn extraction has been verified in real experiments at PS (CERN). Here, we propose a new approach for adiabatic extraction by considering time dependent symplectic maps, where the adiabatic trapping is induced by a modulated dipole or quadrupole kicker. This approach can be particularly relevant when the modulation of the linear tunes is not workable. We discuss the extension of the adiabatic theory results to the symplectic map models and the possible applications to leptonic collider where the dissipation and fluctuations effects are not negligible. We illustrate the results by numerical simulations.

Multi-turn extraction in hadron storage rings using modulated dipole or quadrupole

The adiabatic trapping phenomenon into a nonlinear resonance and the possibility of adiabatic transport in the phase space have been experimentally proven to provide an very efficient extraction mechanism (Multi Turn Extraction (MTE) in high energy hadron storage rings [2, 3]. In the experiments, sextupole and octupole magnets are used to excite a non-linear resonance in transverse phase space [4] and the linear frequencies (linear tunes [4]) are slowly modulated crossing the resonance value and performing the adiabatic trapping of particles. The transverse beam dynamics is described by the one-turn symplectic map (Poincaré map) and the main issues are to extend the adiabatic theory results for Hamiltonian systems to the case of slowly modulated symplectic map in the neighborhood of an elliptic fixed point where the dynamics is almost integrable and the perturbation theory applies. The application to the beam extraction problem requires to optimize and control the trapping efficiency and the quality of the extracted beams [5]. To extend the applicability of this techniques to hadron accelerators, where the change of the linear tunes and the multipolar components in a magnetic lattice can be a complex procedure, we consider the possibility of controlling the adiabatic trapping in the phase space using a dipole or quadrupole 'kicker' in the magnetic lattice whose frequency is slowly modulated to excite a resonance condition with the linear tunes and whose amplitude can be used to optimize the trapping efficiency. The optimization of the MTE is a key issue since the loss of particles in the phase space decreases the luminosity of the beam and create parasitic losses in the accelerator. One of the goal of MTE is to divide the original beam into a fixed number of beamlets with the almost the same number of particle and a defined emittance [1]. The advantage of introducing a modulated kicker is twofold: from one hand it allows to introduce new control parameters in the dynamics that can be easily varied to optimize the efficiency of MTE; from the other hand, the changes of the kicker parameters are easily implemented in a magnetic lattice. The simplest, though non-trivial, model of betatron motion in a hadron storage ring is the 2-dimensional symplectic map

$$\mathcal{M}\begin{pmatrix} q_n \\ p_n \end{pmatrix} = \begin{pmatrix} q_{n+1} \\ p_{n+1} \end{pmatrix} = R(\omega_0)\begin{pmatrix} q_n \\ p_n + k_3 q_n^2 + k_4 q_n^3 + \varepsilon(n)\cos\omega(n)n \end{pmatrix} \tag{1}$$

where $R(\omega_0)$ is the rotation matrix evaluated at the frequency ω_0 (linear tune), n is the turn number and $\varepsilon(n)$ and $\omega(n)$ are the amplitude and the frequency of a dipole kicker that can be adiabatically varied as a function of n. The kicker frequency crosses a resonance value with the linear tune ω_0 (i.e. $\omega = m\omega_0 + \delta$ with δ varying adiabatically in the interval $\delta \in [-\delta_0, \delta_0]$). Using the perturbation theory it is possible to prove that the adiabatic trapping phenomenon in the phase space of the map (1) is described by an interpolating Hamiltonian provided by the Birkhoff Normal Forms Theory

$$\mathcal{H} = \omega_0\, J + \frac{\omega_2}{2}J^2 + \varepsilon\,\mathcal{A}_m J^{3/2}\cos(m\,\theta - \omega t) \tag{2}$$

and to give an explicit correspondence among the parameters of the modulated map (1) and that of the interpolating Hamiltonian (2). According to the results of adiabatic theory [6], we have explicitly computed the trapping probability in the nonlinear resonance for the Hamiltonian system (2) and we have pointed out the dependence of this probability from the kicker parameters. We have numerically checked the applicability of trapping probability estimates in the case of the map (1) (see next section) and that could be extended to more realistic models of the betatron motion defined by 4-dimensional symplectic maps, under suitable conditions.

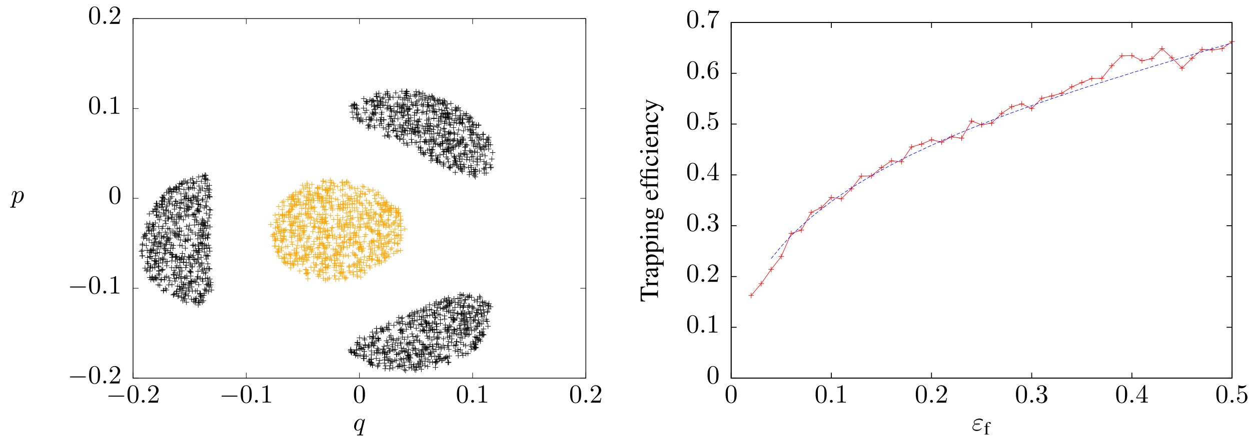

Figure 1: Left: example of the beam splitting obtained by modulating adiabatically the parameters of a dipole element. Right: efficiency of the trapping mechanism as a function of the final dipolar perturbation strength (see Eq. 3) evaluated numerically together with the analytical formula.

Numerical simulations and results

We have performed accurate numerical simulations of the map (1) to study the adiabatic trapping into a 3-order resonance and to check the validity of the trapping probability analytic estimate obtained from the Hamiltonian (2). In Fig. 1 (left) we show an example of the beam splitting obtained modulating adiabatically the parameters of a dipole element according to the following equations

$$\varepsilon(n) = \begin{cases} \varepsilon_{\mathrm{f}}\, n/N & n < N \\ \varepsilon_{\mathrm{f}} & N \leq n \leq 2N \end{cases} \qquad \omega(n) = \begin{cases} \omega_{\mathrm{i}} & n < N \\ \omega_{\mathrm{i}} + (\omega_{\mathrm{f}} - \omega_{\mathrm{i}})(n - N)/N & N \leq n \leq 2N \end{cases} \tag{3}$$

whereas in Fig. 1 (right) we plot the efficiency of the trapping mechanism (i.e. the fraction of particles trapped in the resonance region with respect to the total number of particles in the initial beam given the initial particle distribution) comparing the numerical results with the analytical formulas.
Using the results of the Birkhoff Normal Forms theory for symplectic maps near an elliptic fixed point, we propose a theoretical approach that justifies the use of the interpolating Hamiltonian for a modulated map and point out the applicability conditions of the adiabatic theory results to symplectic maps in a neighborhood of an elliptic fixed point.

Conclusions

The adiabatic trapping into resonance for Hamiltonian systems is a robust phenomenon that can be extended to modulated symplectic maps in the neighborhood of an elliptic point when a resonance condition is satisfied in the phase space. Even if a rigorous proof is not yet available, our results show that it is possible to get estimates of the trapping probability for a particle distribution and to apply the adiabatic theory to realistic model of particle dynamics in hadron colliders. This step is not only relevant in itself, but it is also an essential one in view of applications. Indeed, recently, it has been possible to make adiabatic theory the basis of a successful and novel operational beam manipulation that splits the beam transversely into several beamlets to enable loss-free multi-turn extraction. The new results presented here allow extending the capabilities of performing beam splitting beyond what is know today, thus opening new scenarios for accelerator physics. The ultimate goal of this research is to open the possibility of performing these novel beam manipulations in lepton circular rings.

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